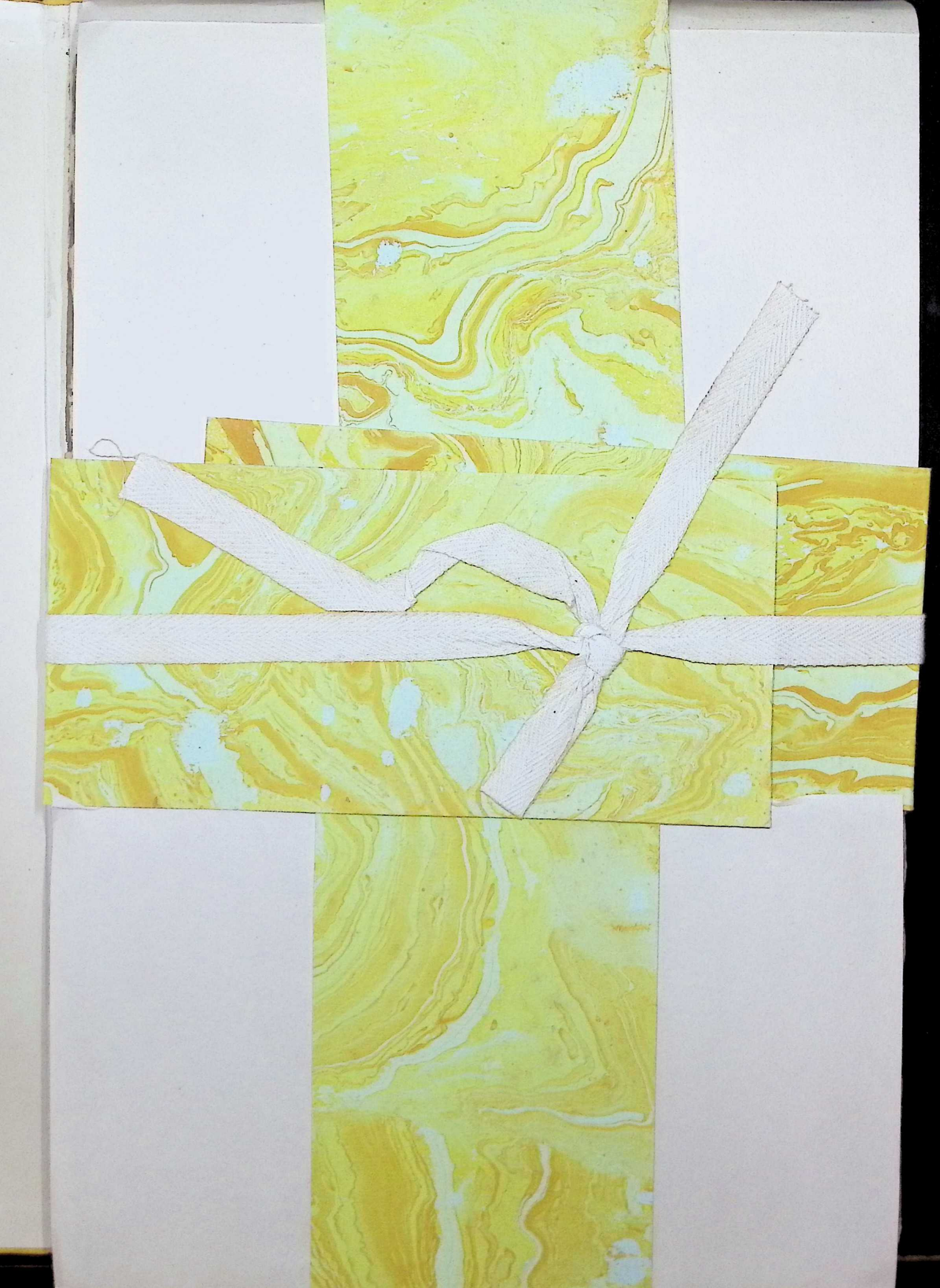
RADHA NATH SIKDAR
MEMORANDUM
BOOK
1853



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Missing Pages

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17 + 18	=	2
31 + 32	=	2
35 to 54	=	20
59 to 62	=	4
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83 to 90	=	8
103 to 106	=	4
177 + 178	=	2
183 to 186	=	4
203 + 204	=	2
207 to 212	=	6
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227 to 238	=	12
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277 to 288	=	12
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From 13 to 354
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Computation of the first 8 powers of

$$u = Ax + Bx^2 + Cx^3 + Dx^4 + Fx^5 + Gx^6 + Hx^7 + Kx^8$$

$$u = Ax + Bx^2 + Cx^3 + Dx^4 + Fx^5 + Gx^6 + Hx^7 + Kx^8$$

$$u^2 = \left[A^2x^2 + 2ABx^3 + 2AC + B^2 \right] x^4 + \left[2AD + 2BC \right] x^5 + \left[2AF + 2BD + C^2 \right] x^6 + \left[2AG + 2BF + 2CD \right] x^7 + \left[2AH + 2BG + 2CF + D^2 \right]$$

$$u^3 = \left[A^3x^3 + 3A^2Bx^4 + 3A^2C + 3AB^2 \right] x^5 + \left[3A^2D + 6ABC + B^3 \right] x^6 + \left[3A^2F + 6ABD + 3AC^2 + 3B^2C \right] x^7 + \left[3A^2G + 6ABF + 6ACD + 3B^2D + 3BC^2 \right] x^8$$

$$u^4 = \left[A^4x^4 + 4A^3Bx^5 + 4A^3C + 6A^2B^2 \right] x^6 + \left[4A^3D + 12A^2BC + 4AB^3 \right] x^7 + \left[4A^3F + 12A^2BD + 6A^2C^2 + 12AB^2C + B^4 \right] x^8$$

$$u^5 = \left[A^5x^5 + 5A^4Bx^6 + 5A^4C + 10A^3B^2 \right] x^7 + \left[5A^4D + 20A^3BC + 10A^2B^3 \right] x^8$$

$$u^6 = \left[A^6x^6 + 6A^5Bx^7 + 6A^5C + 15A^4B^2 \right] x^8$$

$$u^7 = A^7x^7 + 7A^6Bx^8 + \ldots$$

$$u^8 = A^8x^8$$

$$\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^n$$ differentiata

$$fx = \left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)$$

$$f'x = \left[n\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-1} \times \left(B + 2Cx + 3Dx^2 + 4Fx^3 + 5Gx^4 + 6Hx^5 + 7Kx^6\right) \right] =$$

$$f''x = \left[n.\overline{n-1}.\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-2} \times \left(B + 2Cx + 3Dx^2 + 4Fx^3 + 5Gx^4 + 6Hx^5 + 7Kx^6\right)^2 \right]$$

$$+ \left[n\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-1} \times \left(2C + \overline{3.2}\cdot Dx + \overline{4.3}\cdot Fx^2 + \overline{5.4}\, Gx^3 + \overline{6.5}\, Hx^4 + \overline{7.6}\, Kx^5\right) \right]$$

$$f'''x = \left[n.\overline{n-1}.\overline{n-2}\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-3} \times \left(B + 2Cx + 3Dx^2 + 4Fx^3 + 5Gx^4 + 6Hx^5 + 7Kx^6\right)^3 \right]$$

$$+ \left[3n.\overline{n-1}\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-2} \times \left(B + 2Cx + 3Dx^2 + 4Fx^3 + 5Gx^4 + 6Hx^5 + 7Kx^6\right) \times \left(2C + \overline{3.2}\cdot Dx + \overline{4.3}\cdot Fx^2 + \overline{5.4}\, Gx^3 + \overline{6.5}\, Hx^4 + \overline{7.6}\cdot Kx^5\right) \right]$$

$$+ \left[n\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-1} \times \left(\overline{3.2}\cdot D + \overline{4.3.2}\, Fx + \overline{5.4.3}\, Gx^2 + \overline{6.5.4}\, Hx^3 + \overline{7.6.5}\, Kx^4\right) \right]$$

$$f^{w}x = \left[n.\overline{n-1}\cdot\overline{n-2}\cdot\overline{n-3}\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-4} \times \left(B + 2Cx + 3Dx^2 + 4Fx^3 + 5Gx^4 + 6Hx^5 + 7Kx^6\right)^4 \right]$$

$$+ \left[6n.\overline{n-1}\cdot\overline{n-2}\cdot\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-3} \times \left(B + 2Cx + 3Dx^2 + 4Fx^3 + 5Gx^4 + 6Hx^5 + 7Kx^6\right)^2 \times \left(2C + \overline{3\cdot2}\cdot Dx + \overline{4\cdot3}\cdot Fx^2 + \overline{5\cdot4}\cdot Gx^3 + \overline{6\cdot5}\, Hx^4 + 7\cdot6\, Kx^5\right) \right]$$

$$+ \left[3n.\overline{n-1}\cdot\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-2} \times \left(2C + \overline{3\cdot2}.Dx + \overline{4\cdot3}\cdot Fx^2 + \overline{5\cdot4}\cdot Gx^3 + \overline{6\cdot5}\cdot Hx^4 + \overline{7\cdot6}\cdot Kx^5\right)^2 \right]$$

$$+ \left[4n.\overline{n-1}\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-2} \times \left(B + 2Cx + 3Dx^2 + 4Fx^3 + 5Gx^4 + 6Hx^5 + 7Kx^6\right) \times \left(\overline{3\cdot2}\, D + \overline{4\cdot3\cdot2}\, Fx + \overline{5\cdot4\cdot3}\, Gx^2 + \overline{6\cdot5\cdot4}\cdot Hx^3 + 7\cdot6\cdot5\, Kx^4\right) \right]$$

$$+ \left[n\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7\right)^{n-1} \times \left(\overline{4\cdot3\cdot2}\, F + \overline{5\cdot4\cdot3\cdot2}\, Gx + \overline{6\cdot5\cdot4\cdot3}\cdot Hx^2 + \overline{7\cdot6\cdot5\cdot4}\, Kx^3\right) \right]$$

$$f^{n}x = \left\{ n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}\left(A+Bx+Cx^2+Dx^3+Fx^4+Gx^5+Hx^6+Kx^7\right)^{n-5} \right.$$
$$\left. \times\left(B+2Cx+3Dx^2+4Fx^3+5Gx^4+6Hx^5+7Kx^6\right) \right\}$$

$$+\left\{ 10n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\left(A+Bx+Cx^2+Dx^3+Fx^4+Gx^5+Hx^6+Kx^7\right)^{n-4} \right.$$
$$\times\left(B+2Cx+3Dx^2+4Fx^3+5Gx^4+6Hx^5+Kx^6\right)^3$$
$$\left. \times\left(2C+3.2Dx+4.3Fx^2+5.4Gx^3+6.5Hx^4+7.6Kx^5\right) \right\}$$

$$+\left\{ 16n.\overline{n-1}.\overline{n-2}\left(A+Bx+Cx^2+Dx^3+Fx^4+Gx^5+Hx^6+Kx^7+\ldots\right)^{n-3} \right.$$
$$\times\left(B+2Cx+3Dx^2+4Fx^3+5Gx^4+6Hx^5+7Kx^6\right)$$
$$\left. \times\left(2C+3.2Dx+4.3Fx^2+5.4Gx^3+6.5Hx^4+7.6Kx^5\right)^2 \right\}$$

$$+\left\{ 10n.\overline{n-1}.\overline{n-2}.\left(A+Bx+Cx^2+Dx^3+Fx^4+Gx^5+Hx^6+Kx^7\right)^{n-3} \right.$$
$$\times\left(B+2Cx+3Dx^2+4Fx^3+5Gx^4+6Hx^5+7Kx^6\right)^3$$
$$\left. \times\left(3.2D+4.3.2Fx+5.4.3Gx^2+6.5.4Hx^3+7.6.5Kx^4\right) \right\}$$

$$+\left\{ 10n.\overline{n-1}.\left(A+Bx+Cx^2+Dx^3+Fx^4+Gx^5+Hx^6+Kx^7\right)^{n-2} \right.$$
$$\times\left(2C+3.2Dx+4.3Fx^2+5.4Gx^3+6.5Hx^4+7.6Kx^5\right)$$
$$\left. \times\left(3.2D+4.3.2Fx+5.4.3Gx^2+6.5.4Hx^3+7.6.5Kx^4\right) \right\}$$

$$+\left\{\begin{array}{l} n.\overline{n-1}\,(A+Bx+Cx^2+Dx^3+Fx^4+Gx^5+Hx^6+Kx^7)^{n-2} \\ \times\left(\overline{2.1}\,B+\overline{3.2}\,Cx+\overline{4.3.2}\,Dx^2+\overline{5.4.3}\,Fx^3+\overline{6.5.4}\,Gx^4+\overline{7.6.5}\,Hx^5+\overline{8.7.6}\,Kx^6\right)^2 \end{array}\right\}$$

$$+\left\{\begin{array}{l} n.\overline{n-1}\,(A+Bx+Cx^2+Dx^3+Fx^4+Gx^5+Hx^6+Kx^7)^{n-2} \\ \times\left(2C+\overline{3.2}\,Dx+\overline{4.3}\,Fx^2+\overline{5.4}\,Gx^3+\overline{6.5}\,Hx^4+\overline{7.6}\,Kx^5\right) \\ \times\left(\overline{4.3.2}\,F+\overline{5.4.3.2}\,Gx+\overline{6.5.4.3}\,Hx^2+\overline{7.6.5.4}\,Kx^3\right) \end{array}\right\}$$

$$+\left\{\begin{array}{l} n.\overline{n-1}\,(A+Bx+Cx^2+Dx^3+Fx^4+Gx^5+Hx^6+Kx^7)^{n-2} \\ \times\left(B+2Cx+3Dx^2+4Fx^3+5Gx^4+6Hx^5+7Kx^6\right) \\ \times\left(\overline{5.4.3.2}\,G+\overline{6.5.4.3.2}\,Hx+\overline{7.6.5.4.3}\,Kx^2\right) \end{array}\right\}$$

$$+\left\{\begin{array}{l} n.(A+Bx+Cx^2+Dx^3+Fx^4+Gx^5+Hx^6+Kx^7)^{n-1} \\ \times\left(\overline{6.5.4.3.2}\,H+\overline{7.6.5.4.3}\,Kx+\ldots\right) \end{array}\right\}$$

$$f. = A^n$$

$$f.' = n A^{n-1} . B$$

$$f.'' = n.\overline{n-1} . A^{n-2} . B^2 + 2n . A^{n-1} . C$$

$$f.''' = n.\overline{n-1}.\overline{n-2} . A^{n-3} . B^3 + \overline{2.3}\, n.\overline{n-1} . A^{n-2} . B.C + \overline{2.3}\, n A^{n-1} . D$$

$$f.'''' = n.\overline{n-1}.\overline{n-2}.\overline{n-3} . A^{n-4} . B^4 + \overline{2.2.3}\, n.\overline{n-1}.\overline{n-2} A^{n-3} . B^2 . C + \overline{2.2.3}\, n.\overline{n-1} A^{n-2} . C^2 + \overline{2.3.4} . n.\overline{n-1} . A^{n-2} . B D + \overline{2.3.4} . n A^{n-1} . F$$

$$f.^{v} = n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4} . A^{n-5} . B^5 + \overline{2.2.5} . n.\overline{n-1}.\overline{n-2}.\overline{n-3} . A^{n-4} B^3 . C + \overline{2.2.3.5} . n.\overline{n-1}.\overline{n-2} . A^{n-3} . B . C^2 + \overline{2.2.3.5} . n.\overline{n-1}.\overline{n-2} . A^{n-3} . B^2 . D + \overline{2.2.2.3.5} . n.\overline{n-1} . A^{n-2} . C . D + \overline{2.3.4.5} . n.\overline{n-1} . A^{n-2} . B . F + \overline{2.3.4.5} . n A^{n-1} . G$$

$$f.^{vi} = n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}.\overline{n-5} . A^{n-6} . B^6 + \overline{2.3.5} . n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4} . A^{n-5} . B^4 . C + \overline{2.2.2.3.5} . n.\overline{n-1}.\overline{n-2}.\overline{n-3} . A^{n-4} . B^2 . C^2 + \overline{2.3.4.5} . n.\overline{n-1}.\overline{n-2}.\overline{n-3} . A^{n-4} . B^3 . D + \overline{2.3.4.5} . n.\overline{n-1}.\overline{n-2} . A^{n-3} . C^3 + \overline{2.2.3.4.5} . n.\overline{n-1}.\overline{n-2} . A^{n-3} . B C . D + \overline{2.3.3.4.5} . n.\overline{n-1}.\overline{n-2} . A^{n-3} . B^2 . F + \overline{2.2.2.4.5} . n.\overline{n-1} . A^{n-2} . D^2 + \overline{2.2.3.4.5} . n.\overline{n-1} . A^{n-2} . C . F + \overline{2.3.4.5.6} . n.\overline{n-1} . A^{n-2} . B . G + \overline{2.3.4.5.6} . n A^{n-1} . H$$

Let $x = 0$

$u = A$

$\frac{du}{dx} = B$

$\frac{d^2u}{dx^2} = 2C$

$\frac{d^3u}{dx^3} = \overline{2.3} D$

$\frac{d^4u}{dx^4} = \overline{2.3.4} . F$

$\frac{d^5u}{dx^5} = \overline{2.3.4.5} G$

$\frac{d^6u}{dx^6} = \overline{2.3.4.5.6} H$

u^n Differentiata

$$d u^n = n u^{n-1} \cdot du$$ Exp.

$$d^2 u^n = n \cdot \overline{n-1} \cdot u^{n-2} \cdot du^2 + n u^{n-1} \cdot d^2 u$$ Ex. 2.

$$d^3 u^n = n \cdot \overline{n-1} \cdot \overline{n-2} \cdot u^{n-3} \cdot du^3 + 3 n \cdot \overline{n-1} \cdot u^{n-2} \cdot du \cdot d^2 u + n u^{n-1} \cdot d^3 u$$ Ex. 3.

$$d^4 u^n = n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot u^{n-4} \cdot du^4 + 6 n \cdot \overline{n-1} \cdot \overline{n-2} \cdot u^{n-3} \cdot du^2 \cdot d^2 u + 3 n \cdot \overline{n-1} \cdot u^{n-2} \cdot d^2 u^2 + 4 n \cdot \overline{n-1} \cdot u^{n-2} \cdot du \cdot d^3 u + n u^{n-1} \cdot d^4 u$$ Ex. 4.

$$d^5 u^n = n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot \overline{n-4} \cdot u^{n-5} \cdot du^5 + 10 n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot u^{n-4} \cdot du^3 \cdot d^2 u + 15 \cdot n \cdot \overline{n-1} \cdot \overline{n-2} \cdot u^{n-3} \cdot du \cdot d^2 u^2 + 10 \cdot n \cdot \overline{n-1} \cdot \overline{n-2} \cdot u^{n-3} \cdot du^2 d^3 u + 10 n \cdot \overline{n-1} \cdot u^{n-2} d^2 u \, d^3 u + 5 n \cdot \overline{n-1} \cdot u^{n-2} \cdot du \cdot d^4 u + n u^{n-1} \cdot d^5 u$$ Ex. 5.

$$d^6 u^n = n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot \overline{n-4} \cdot \overline{n-5} \cdot u^{n-6} \cdot du^6 + 15 n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot \overline{n-4} \cdot u^{n-5} \cdot du^4 \cdot d^2 u + 45 \cdot n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot u^{n-4} \cdot du^2 d^2 u^2 + 20 n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot u^{n-4} \cdot du^3 \cdot d^3 u + 15 n \cdot \overline{n-1} \cdot \overline{n-2} \cdot u^{n-3} \cdot d^2 u^3 + 60 n \cdot \overline{n-1} \cdot \overline{n-2} \cdot u^{n-3} \cdot du \cdot d^2 u \cdot d^3 u + 15 n \cdot \overline{n-1} \cdot \overline{n-2} \cdot u^{n-3} \cdot du^2 \cdot d^4 u + 10 n \cdot \overline{n-1} \cdot u^{n-2} \cdot d^3 u^2 + 15 n \cdot \overline{n-1} \cdot u^{n-2} \cdot d^2 u \cdot d^4 u + 6 n \cdot \overline{n-1} \cdot u^{n-2} \cdot du \cdot d^5 u + n u^{n-1} \cdot d^6 u$$

$$u = A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6 + Kx^7$$

$$\frac{du}{dx} = B + 2Cx + 3Dx^2 + 4Fx^3 + 5Gx^4 + 6Hx^5 + 7Kx^6$$

$$\frac{d^2u}{dx^2} = 2C + \overline{3 \cdot 2} Dx + \overline{4 \cdot 3} Fx^2 + \overline{5 \cdot 4} \cdot Gx^3 + \overline{6 \cdot 5} Hx^4 + \overline{7 \cdot 6} \cdot Kx^5$$

$$\frac{d^3u}{dx^3} = \overline{3 \cdot 2} D + \overline{4 \cdot 3 \cdot 2} Fx + \overline{5 \cdot 4 \cdot 3} Gx^2 + \overline{6 \cdot 5 \cdot 4} Hx^3 + \overline{7 \cdot 6 \cdot 5} Kx^4$$

$$\frac{d^4u}{dx^4} = \overline{4 \cdot 3 \cdot 2} F + \overline{5 \cdot 4 \cdot 3 \cdot 2} Gx + \overline{6 \cdot 5 \cdot 4 \cdot 3} Hx^2 + \overline{7 \cdot 6 \cdot 5 \cdot 4} Kx^3$$

$$\frac{d^5u}{dx^5} = \overline{5 \cdot 4 \cdot 3 \cdot 2} G + \overline{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} Hx + \overline{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3} Kx^2$$

$$\frac{d^6u}{dx^6} = \overline{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} H + \overline{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3} Kx$$

$$\left(A + Bx + Cx^2 + Dx^3 + Fx^4 + Gx^5 + Hx^6\right)^n =$$

$$\left.\begin{aligned} & A^n + n A^{n-1} B x + \frac{n.n-1}{1.2} A^{n-2} B^2 \\ & \qquad + n A^{n-1} C \end{aligned}\right\} x^2 + \left.\begin{aligned} & \frac{n.n-1.n-2}{1.2.3} A^{n-3} B^3 \\ & + n.n-1 A^{n-2} B C \\ & + n A^{n-1} D \end{aligned}\right\} x^3$$

$$\left.\begin{aligned} & + \frac{n.n-1.n-2.n-3}{1.2.3.4} A^{n-4} B^4 \\ & + \frac{n.n-1.n-2}{1.2} A^{n-3} B^2 C \\ & + \frac{n.n-1}{1.2} A^{n-2} C^2 \\ & + n.n-1 A^{n-2} B D \\ & + n A^{n-1} F \end{aligned}\right\} x^4 + \left.\begin{aligned} & \frac{n.n-1.n-2.n-3.n-4}{1.2.3.4.5} A^{n-5} B^5 \\ & + \frac{n.n-1.n-2.n-3}{1.2.3} A^{n-4} B^3 C \\ & + \frac{n.n-1.n-2}{1.2} A^{n-3} B C^2 \\ & + \frac{n.n-1.n-2}{1.2} A^{n-3} B^2 D \\ & + n.n-1 A^{n-2} C D \\ & + n.n-1 A^{n-2} B F \\ & + n A^{n-1} G \end{aligned}\right\} x^5$$

$$+\frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}.\overline{n-5}}{1.2.3.4.5.6}A^{n-6}.B^6$$

$$+\frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}}{1.2.3.4}A^{n-5}.B^4C^1$$

$$+\frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}}{1.2.3}A^{n-4}.B^2C^2$$

$$+\frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}}{1.6}A^{n-4}.B^3D$$

$$+\frac{n.\overline{n-1}.\overline{n-2}}{1.6}A^{n-3}.C^3$$

$$+n.\overline{n-1}.\overline{n-2}.A^{n-3}.B.C.D$$

$$+\frac{n.\overline{n-1}.\overline{n-2}}{2}A^{n-3}.B^2F$$

$$+\frac{n.\overline{n-1}}{2}A^{n-2}.D^2$$

$$+n.\overline{n-1}.A^{n-2}.CF$$

$$+n.\overline{n-1}.A^{n-2}.BG$$

$$n.A^{n-1}.H$$

Let $\alpha y + \beta y^2 + \gamma y^3 + \delta y^4 + \varepsilon y^5 + \zeta y^6 + \theta y^7 + \lambda y^8 = \mathfrak{A}x + \mathfrak{B}x^2 + \mathfrak{C}x^3 + \mathfrak{D}x^4 + \mathfrak{E}x^5 + \mathfrak{F}x^6 + \mathfrak{G}x^7 + \mathfrak{H}x^8 + \ldots\ldots$

Required $y = Ax + Bx^2 + Cx^3 + Dx^4 + Ex^5 + Fx^6 + Gx^7 + Hx^8 + \ldots\ldots$

$\alpha y =$ αAx $+\alpha B.x^2$ $+\alpha Cx^3$ $+\alpha Dx^4$ $+\alpha Fx^6$ $+\alpha Gx^7$ $+\alpha Hx^8$

$+\beta y^2 =$ $+\beta A^2.x^2$ $+2\beta AB.x^3$ $+2\beta AC\,x^4 + \beta B^2$ $+2\beta AD\,x^5 + 2\beta BC$ $+2\beta AE\,x^6 + 2\beta BD + \beta C^2$ $+2\beta AG\,x^7 + 2\beta BF + 2\beta CD$ $+2\beta AH\,x^8 + 2\beta BG + 2\beta CF + \beta D^2$

$+\gamma y^3 =$ $+\gamma A^3.x^3$ $+3\gamma A^2B\,x^4$ $+3\gamma A^2C\,x^5 + 3\gamma AB^2$ $+3\gamma A^2D\,x^6 + 6\gamma ABC + \gamma B^3$ $+3\gamma A^2F\,x^7 + 6\gamma ABD + 3\gamma AC^2 + 3\gamma B^2C$ $+3\gamma A^2G\,x^8 + 6\gamma ABF + 6\gamma ACD + 3\gamma B^2D + 3\gamma BC^2$

$+\delta y^4 =$ $+\delta A^4.x^4$ $+4\delta A^3Bx^5$ $+4\delta A^3C\,x^6 + 6\delta A^2B^2$ $+4\delta A^3D\,x^7 + 12\delta A^2BC + 4\delta AB^3$ $+4\delta A^3E\,x^8 + 12\delta A^2BD + 6\delta A^2C^2 + 12\delta AB^2C + \delta B^4$

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$$+\delta y^4 = +\delta A^4 x^4 + 4\delta A^3 B x^3 + 6\delta A^2 B^2 x^2 + 4\delta A B^3 x + \delta B^4$$
$$+4\delta A^3 D \quad +12\delta A^2 BD \quad +12\delta A B^2 D \quad +4\delta B^3 D$$
$$+4\delta A^3 C \quad +12\delta A^2 BC \quad +6\delta A B^2 C$$
$$+4\delta A^3 B \quad +6\delta A^2 B^2$$
$$+4\delta A^3 E \quad +12\delta A^2 BE \quad +6\delta A B^2 E \quad +4\delta B^3 E$$
$$+\delta A^2 D^2 \quad +4\delta A B D^2 \quad +2\delta B^2 D^2$$
$$+2\delta A^2 CD \quad +4\delta A B CD \quad +2\delta B^2 CD$$
$$+\delta A^2 C^2 \quad +2\delta A B C^2 \quad +\delta B^2 C^2$$
$$+2\delta A^2 BD \quad +4\delta A B^2 D \quad +2\delta B^3 D$$
$$+2\delta A^2 BC \quad +2\delta A B^2 C$$
$$+\delta A^2 B^2 \quad +\delta A B^3$$

$$+\varepsilon y^5 = +\varepsilon A^5 x^5 + 5\varepsilon A^4 B x^4 + 10\varepsilon A^3 B^2 x^3 + 10\varepsilon A^2 B^3 x^2 + 5\varepsilon A B^4 x + \varepsilon B^5$$
$$+5\varepsilon A^4 D \quad +20\varepsilon A^3 BD \quad +30\varepsilon A^2 B^2 D \quad +20\varepsilon A B^3 D \quad +5\varepsilon B^4 D$$
$$+5\varepsilon A^4 C \quad +20\varepsilon A^3 BC \quad +15\varepsilon A^2 B^2 C \quad +5\varepsilon A B^3 C$$
$$+5\varepsilon A^4 B \quad +10\varepsilon A^3 B^2 \quad +5\varepsilon A^2 B^3$$
$$+10\varepsilon A^3 D^2 \quad +30\varepsilon A^2 BD^2 \quad +30\varepsilon A B^2 D^2 \quad +10\varepsilon B^3 D^2$$
$$+10\varepsilon A^3 CD \quad +30\varepsilon A^2 BCD \quad +20\varepsilon A B^2 CD \quad +5\varepsilon B^3 CD$$
$$+5\varepsilon A^3 C^2 \quad +15\varepsilon A^2 BC^2 \quad +10\varepsilon A B^2 C^2 \quad +\varepsilon B^3 C^2$$
$$+10\varepsilon A^3 BD \quad +30\varepsilon A^2 B^2 D \quad +20\varepsilon A B^3 D \quad +5\varepsilon B^4 D$$
$$+10\varepsilon A^3 BC \quad +20\varepsilon A^2 B^2 C \quad +5\varepsilon A B^3 C$$
$$+5\varepsilon A^3 B^2 \quad +5\varepsilon A^2 B^3$$
$$+10\varepsilon A^2 D^3 \quad +30\varepsilon A B D^3 \quad +10\varepsilon B^2 D^3$$
$$+10\varepsilon A^2 CD^2 \quad +30\varepsilon A B CD^2 \quad +10\varepsilon B^2 CD^2$$
$$+5\varepsilon A^2 C^2 D \quad +15\varepsilon A B C^2 D \quad +5\varepsilon B^2 C^2 D$$
$$+\varepsilon A^2 C^3 \quad +3\varepsilon A B C^3 \quad +\varepsilon B^2 C^3$$
$$+10\varepsilon A^2 BD^2 \quad +30\varepsilon A B^2 D^2 \quad +10\varepsilon B^3 D^2$$
$$+10\varepsilon A^2 BCD \quad +20\varepsilon A B^2 CD \quad +5\varepsilon B^3 CD$$
$$+5\varepsilon A^2 BC^2 \quad +5\varepsilon A B^2 C^2 \quad +\varepsilon B^3 C^2$$
$$+10\varepsilon A^2 B^2 D \quad +10\varepsilon A B^3 D \quad +5\varepsilon B^4 D$$
$$+10\varepsilon A^2 B^2 C \quad +10\varepsilon A B^3 C$$
$$+5\varepsilon A^2 B^3 \quad +5\varepsilon A B^4$$
$$+\varepsilon A B^5$$

$$+\zeta y^6 = +\zeta A^6 x^6 + 6\zeta A^5 B x^5 + 15\zeta A^4 B^2 x^4 + 20\zeta A^3 B^3 x^3 + 15\zeta A^2 B^4 x^2 + 6\zeta A B^5 x + \zeta B^6$$
$$+6\zeta A^5 D \quad +30\zeta A^4 BD \quad +60\zeta A^3 B^2 D \quad +60\zeta A^2 B^3 D \quad +30\zeta A B^4 D \quad +6\zeta B^5 D$$
$$+6\zeta A^5 C \quad +30\zeta A^4 BC \quad +60\zeta A^3 B^2 C \quad +30\zeta A^2 B^3 C \quad +6\zeta A B^4 C$$
$$+6\zeta A^5 B \quad +15\zeta A^4 B^2 \quad +20\zeta A^3 B^3 \quad +15\zeta A^2 B^4 \quad +6\zeta A B^5$$
$$+15\zeta A^4 D^2 \quad +120\zeta A^3 BD^2 \quad +180\zeta A^2 B^2 D^2 \quad +120\zeta A B^3 D^2 \quad +30\zeta B^4 D^2$$
$$+15\zeta A^4 CD \quad +120\zeta A^3 BCD \quad +180\zeta A^2 B^2 CD \quad +60\zeta A B^3 CD \quad +15\zeta B^4 CD$$
$$+15\zeta A^4 C^2 \quad +60\zeta A^3 BC^2 \quad +60\zeta A^2 B^2 C^2 \quad +15\zeta A B^3 C^2 \quad +\zeta B^4 C^2$$
$$+15\zeta A^4 BD \quad +60\zeta A^3 B^2 D \quad +60\zeta A^2 B^3 D \quad +15\zeta A B^4 D \quad +6\zeta B^5 D$$
$$+15\zeta A^4 BC \quad +30\zeta A^3 B^2 C \quad +15\zeta A^2 B^3 C \quad +\zeta A B^4 C$$
$$+6\zeta A^4 B^2 \quad +15\zeta A^3 B^3 \quad +6\zeta A^2 B^4$$
$$+20\zeta A^3 D^3 \quad +180\zeta A^2 BD^3 \quad +180\zeta A B^2 D^3 \quad +30\zeta B^3 D^3$$
$$+20\zeta A^3 CD^2 \quad +180\zeta A^2 BCD^2 \quad +120\zeta A B^2 CD^2 \quad +15\zeta B^3 CD^2$$
$$+15\zeta A^3 C^2 D \quad +90\zeta A^2 BC^2 D \quad +60\zeta A B^2 C^2 D \quad +6\zeta B^3 C^2 D$$
$$+6\zeta A^3 C^3 \quad +30\zeta A^2 BC^3 \quad +15\zeta A B^2 C^3 \quad +\zeta B^3 C^3$$
$$+20\zeta A^3 BD^2 \quad +90\zeta A^2 B^2 D^2 \quad +60\zeta A B^3 D^2 \quad +15\zeta B^4 D^2$$
$$+20\zeta A^3 BCD \quad +120\zeta A^2 B^2 CD \quad +60\zeta A B^3 CD \quad +15\zeta B^4 CD$$
$$+15\zeta A^3 BC^2 \quad +60\zeta A^2 B^2 C^2 \quad +30\zeta A B^3 C^2 \quad +6\zeta B^4 C^2$$
$$+15\zeta A^3 B^2 D \quad +60\zeta A^2 B^3 D \quad +30\zeta A B^4 D \quad +6\zeta B^5 D$$
$$+15\zeta A^3 B^2 C \quad +30\zeta A^2 B^3 C \quad +6\zeta A B^4 C$$
$$+6\zeta A^3 B^3 \quad +15\zeta A^2 B^4 \quad +6\zeta A B^5$$
$$+15\zeta A^2 D^4 \quad +120\zeta A B D^4 \quad +30\zeta B^2 D^4$$
$$+15\zeta A^2 CD^3 \quad +120\zeta A B CD^3 \quad +30\zeta B^2 CD^3$$
$$+6\zeta A^2 C^2 D^2 \quad +60\zeta A B C^2 D^2 \quad +15\zeta B^2 C^2 D^2$$
$$+\zeta A^2 C^3 D \quad +30\zeta A B C^3 D \quad +6\zeta B^2 C^3 D$$
$$+\zeta A^2 C^4 \quad +6\zeta A B C^4 \quad +\zeta B^2 C^4$$
$$+15\zeta A^2 BD^3 \quad +120\zeta A B^2 D^3 \quad +30\zeta B^3 D^3$$
$$+15\zeta A^2 BCD^2 \quad +120\zeta A B^2 CD^2 \quad +30\zeta B^3 CD^2$$
$$+6\zeta A^2 BC^2 D \quad +60\zeta A B^2 C^2 D \quad +15\zeta B^3 C^2 D$$
$$+\zeta A^2 BC^3 \quad +30\zeta A B^2 C^3 \quad +6\zeta B^3 C^3$$
$$+15\zeta A^2 B^2 D^2 \quad +120\zeta A B^3 D^2 \quad +30\zeta B^4 D^2$$
$$+15\zeta A^2 B^2 CD \quad +120\zeta A B^3 CD \quad +30\zeta B^4 CD$$
$$+6\zeta A^2 B^2 C^2 \quad +60\zeta A B^3 C^2 \quad +15\zeta B^4 C^2$$
$$+6\zeta A^2 B^3 D \quad +30\zeta A B^4 D \quad +6\zeta B^5 D$$
$$+6\zeta A^2 B^3 C \quad +30\zeta A B^4 C$$
$$+15\zeta A^2 B^4 \quad +15\zeta A B^5$$
$$+\zeta A B^6$$

$$\alpha A = \text{क}, \qquad A = \frac{\text{क}}{\alpha}$$

$$\alpha B + \beta A = \text{ख}, \qquad \alpha B + \frac{\beta \text{क}}{\alpha} = \text{ख}, \qquad B = \frac{\text{ख}}{\alpha} - \frac{\beta \text{क}}{\alpha^2}$$

$$B^2 = \frac{\text{ख}^2}{\alpha^2} - \frac{2\beta \text{क} \text{ख}}{\alpha^3} + \frac{\beta^2 \text{क}^2}{\alpha^4}, \qquad B^2 = \frac{\text{ख}^2}{\alpha^2} - \frac{2\beta \text{क}^2 \text{ख}}{\alpha^3} + \frac{2\beta^2 \text{क}^4 \text{ख}}{\alpha^4} - \frac{\beta^2 \text{क}^4}{\alpha^4}$$

$$\alpha C + 2\beta AB + \gamma A^2 = \text{ग}, \qquad \alpha C + \frac{2\text{क}}{\alpha}\left(\frac{\text{ख}}{\alpha} - \frac{\beta \text{क}^2}{\alpha^3}\right) + \frac{\gamma \text{क}^2}{\alpha^2} = \text{ग}$$

$$C = \frac{\text{ग}}{\alpha} - \frac{2\text{क}\text{ख}}{\alpha^2} + \frac{2\beta \text{क}^3}{\alpha^3} - \frac{\gamma \text{क}^2}{\alpha^3}$$

$$C^2 = \frac{\text{ग}^2}{\alpha^2} + \frac{4\text{क}^2\text{ख}^2}{\alpha^4} + \frac{4\beta^2\text{क}^6}{\alpha^6} - \frac{\gamma^2\text{क}^4}{\alpha^6} - \frac{4\text{क}\text{ख}\text{ग}}{\alpha^3} + \frac{4\beta\text{क}^3\text{ग}}{\alpha^4} - \frac{2\gamma\text{क}^3\text{ग}}{\alpha^5} - \frac{2\beta\text{क}^4\text{ख}}{\alpha^5} + \frac{4\gamma\text{क}^4\text{ख}}{\alpha^6} - \frac{4\beta\gamma\text{क}^5}{\alpha^6}$$

$$\alpha D + 2\beta AC + \beta B^2 + 3\gamma A^2 B + \delta A^4 = \text{घ}$$

$$\alpha D + \frac{2\beta \cdot \text{क}}{\alpha}\left(\frac{\text{ग}}{\alpha} - \frac{2\text{क}\text{ख}}{\alpha^3} + \frac{2\beta\text{क}^3}{\alpha^5} - \frac{\gamma\text{क}^3}{\alpha^4}\right) + \beta\left(\frac{\text{ख}^2}{\alpha^2} - \frac{2\beta\text{क}^2\text{ख}}{\alpha^4} + \frac{\beta^2\text{क}^4}{\alpha^6}\right) + \frac{3\gamma\text{क}^2}{\alpha^2}\left(\frac{\text{ख}}{\alpha} - \frac{\beta\text{क}^2}{\alpha^3}\right) + \frac{\delta\text{क}^4}{\alpha^4} = \text{घ}$$

$$\alpha D + \frac{2\beta\text{क}\text{ग}}{\alpha^2} - \frac{4\beta\text{क}^2\text{ख}}{\alpha^4} + \frac{4\beta^2\text{क}^4}{\alpha^6} - \frac{2\beta\gamma\text{क}^4}{\alpha^6} + \frac{\beta\text{ख}^2}{\alpha^2} - \frac{2\beta^2\text{क}^2\text{ख}}{\alpha^4} + \frac{\beta^3\text{क}^4}{\alpha^6} + \frac{3\gamma\text{क}^2\text{ख}}{\alpha^3} - \frac{3\beta\gamma\text{क}^4}{\alpha^5} + \frac{\delta\text{क}^4}{\alpha^4} = \text{घ}$$

$$D = \left\{\begin{array}{l} \frac{\text{घ}}{\alpha} - \frac{2\beta\text{क}\text{ग}}{\alpha^3} + \frac{4\beta\text{क}^2\text{ख}}{\alpha^5} - \frac{4\beta^2\text{क}^4}{\alpha^7} + \frac{5\beta\gamma\text{क}^4}{\alpha^6} - \frac{\beta\text{ख}^2}{\alpha^3} \\ - \frac{2\beta^2\text{क}^2\text{ख}}{\alpha^5} - \frac{\beta^3\text{क}^4}{\alpha^6} - \frac{3\gamma\text{क}^2\text{ख}}{\alpha^4} + \frac{3\beta\gamma\text{क}^4}{\alpha^6} - \frac{\delta\text{क}^4}{\alpha^6} \end{array}\right.$$

$$\alpha C + 2\beta AB + \gamma A^3 = \pi \quad ; \quad \alpha C + \frac{2\beta\kappa}{\alpha}\left(\frac{\kappa}{\alpha} - \frac{\beta\kappa^2}{\alpha^3}\right) + \frac{\gamma\kappa^3}{\alpha^3} = \pi$$

$$\alpha C + \frac{2\beta\kappa\kappa}{\alpha^2} - \frac{2\beta^2\kappa^3}{\alpha^4} + \frac{\gamma\kappa^3}{\alpha^3} = \pi$$

$$C = \frac{\pi}{\alpha} - \frac{2\beta\kappa\kappa}{\alpha^3} - \frac{\gamma\kappa^3}{\alpha^4} + \frac{2\beta^2\kappa^3}{\alpha^5}$$

$$C^2 = \left\{ \begin{array}{l} \frac{\pi^2}{\alpha^2} - \frac{4\beta\kappa\kappa\pi}{\alpha^4} - \frac{2\gamma\kappa^3\pi}{\alpha^5} + \frac{4\beta^2\kappa^2\kappa^2}{\alpha^6} + \frac{4\beta^2\kappa^3\pi}{\alpha^6} \\ + \frac{4\beta\gamma\kappa^4\kappa}{\alpha^7} + \frac{\gamma^2\kappa^6}{\alpha^8} - \frac{8\beta^3\kappa^4\kappa}{\alpha^8} - \frac{4\beta^2\gamma\kappa^6}{\alpha^9} + \frac{4\beta^4\kappa^6}{\alpha^{10}} \end{array} \right.$$

$$BC = - \left\{ \begin{array}{l} \frac{\kappa\pi}{\alpha^2} - \frac{2\beta\kappa\kappa^2}{\alpha^4} - \frac{\beta\kappa^2\pi}{\alpha^4} \\ - \frac{\gamma\kappa^3\kappa}{\alpha^5} + \frac{4\beta^2\kappa^3\kappa}{\alpha^6} + \frac{\beta\gamma\kappa^5}{\alpha^7} - \frac{2\beta^3\kappa^5}{\alpha^8} \end{array} \right.$$

$$\alpha D + 2\beta AC + \beta B^2 + 3\gamma A^2 B + \delta A^4 = \nu$$

$$\alpha D + 2\beta\frac{\kappa}{\alpha}\left(\frac{\pi}{\alpha} - \frac{2\beta\kappa\kappa}{\alpha^3} - \frac{\gamma\kappa^3}{\alpha^4} + \frac{2\beta^2\kappa^3}{\alpha^5}\right)$$

$$+ \beta\left(\frac{\kappa^2}{\alpha^2} - \frac{2\beta\kappa^2\kappa}{\alpha^4} + \frac{\beta^2\kappa^4}{\alpha^6}\right) + 3\gamma\frac{\kappa^2}{\alpha^2}\left(\frac{\kappa}{\alpha} - \frac{\beta\kappa^2}{\alpha^3}\right) + \delta\frac{\kappa^4}{\alpha^4} = \nu$$

$$\alpha D + \frac{2\beta\kappa\pi}{\alpha^2} + \frac{\beta\kappa^2}{\alpha^2} + \frac{3\gamma\kappa^2\kappa}{\alpha^3} - \frac{5\beta^2\kappa^2\kappa}{\alpha^4}$$

$$+ \frac{\delta\kappa^4}{\alpha^4} - \frac{5\beta\gamma\kappa^4}{\alpha^5} + \frac{5\beta^3\kappa^4}{\alpha^6} = \nu$$

$$D = \left\{ \begin{array}{l} \frac{\nu}{\alpha} - \frac{2\beta\kappa\pi}{\alpha^3} - \frac{\beta\kappa^2}{\alpha^3} - \frac{3\gamma\kappa^2\kappa}{\alpha^4} \\ + \frac{5\beta^2\kappa^2\kappa}{\alpha^5} - \frac{\delta\kappa^4}{\alpha^5} + \frac{5\beta\gamma\kappa^4}{\alpha^6} - \frac{5\beta^3\kappa^4}{\alpha^7} \end{array} \right.$$

$$BD = \left\{ \begin{array}{l} \frac{\text{ख च}}{\alpha^2} - \frac{2\beta\text{क ख ग}}{\alpha^4} - \frac{\beta\text{ख}^3}{\alpha^4} - \frac{\beta\text{क}^2\text{च}}{\alpha^4} - \frac{3\gamma\text{क}^2\text{ख}^2}{\alpha^6} + \frac{7\beta^2\text{क}^2\text{ख}^2}{\alpha^6} \\ - \frac{\delta\text{क}^4\text{ख}}{\alpha^6} + \frac{2\beta^2\text{क}^3\text{ग}}{\alpha^6} + \frac{8\beta\gamma\text{क}^4\text{ख}}{\alpha^7} - \frac{11\beta^3\text{क}^4\text{ख}}{\alpha^8} + \frac{\beta\delta\text{क}^6}{\alpha^8} + \\ - \frac{5\beta^2\gamma\text{क}^6}{\alpha^9} + \frac{5\beta^4\text{क}^6}{\alpha^{10}} \end{array} \right.$$

$$\alpha\text{च} + 2\beta AD + 2\beta BC + 3\gamma A^2C + 3\gamma AB^2 + 4\delta A^3B + 8A^5 = \text{ख}$$

Now

$$2\beta AD = 2\beta\frac{\text{क}}{\alpha}\left(\frac{\text{च}}{\alpha} - \frac{2\beta\text{क ग}}{\alpha^3} - \frac{\beta\text{ख}^2}{\alpha^3} - \frac{3\gamma\text{क}^2\text{ख}}{\alpha^4} + \frac{6\beta^2\text{क}^2\text{ख}}{\alpha^5} - \frac{\delta\text{क}^4}{\alpha^5} + \frac{5\beta\gamma\text{क}^4}{\alpha^6} - \frac{5\beta^3\text{क}^4}{\alpha^7}\right) =$$

$$\frac{2\beta\text{क च}}{\alpha^2} - \frac{4\beta^2\text{क}^2\text{ग}}{\alpha^4} - \frac{2\beta^2\text{क ख}^2}{\alpha^4} - \frac{6\beta\gamma\text{क}^3\text{ख}}{\alpha^5} + \frac{12\beta^3\text{क}^3\text{ख}}{\alpha^6} - \frac{2\beta\delta\text{क}^5}{\alpha^6} + \frac{10\beta^2\gamma\text{क}^5}{\alpha^7} - \frac{10\beta^4\text{क}^5}{\alpha^8}$$

Also,

$$+2\beta BC = 2\beta\left(\frac{\text{ख ग}}{\alpha^2} - \frac{2\beta\text{क ख}^2}{\alpha^4} - \frac{\beta\text{क}^2\text{ग}}{\alpha^4} - \frac{\gamma\text{क}^3\text{ख}}{\alpha^5} + \frac{4\beta^2\text{क}^3\text{ख}}{\alpha^6} + \frac{\beta\gamma\text{क}^5}{\alpha^7} - \frac{2\beta^3\text{क}^5}{\alpha^8}\right) = \frac{2\beta\text{ख ग}}{\alpha^2} - \frac{4\beta^2\text{क ख}^2}{\alpha^4} - \frac{2\beta^2\text{क}^2\text{ग}}{\alpha^4} - \frac{2\beta\gamma\text{क}^3\text{ख}}{\alpha^5} + \frac{8\beta^3\text{क}^3\text{ख}}{\alpha^6} + \frac{2\beta^2\gamma\text{क}^5}{\alpha^7} - \frac{4\beta^4\text{क}^5}{\alpha^8}$$

Also

$$3\gamma A^2C = 3\gamma\frac{\text{क}^2}{\alpha^2}\left(\frac{\text{ग}}{\alpha} - \frac{2\beta\text{क ख}}{\alpha^5} - \frac{\gamma\text{क}^3}{\alpha^4} + \frac{2\beta^2\text{क}^3}{\alpha^5}\right) =$$

$$\frac{3\gamma\text{क}^2\text{ग}}{\alpha^3} - \frac{6\beta\gamma\text{क}^2\text{ख}}{\alpha^5} - \frac{3\gamma^2\text{क}^5}{\alpha^6} + \frac{6\beta^2\gamma\text{क}^5}{\alpha^7}$$

Likewise

$$3\gamma AB^2 = 3\gamma\frac{\text{क}}{\alpha}\left(\frac{\text{ख}^2}{\alpha^2} - \frac{2\beta\text{क}^2\text{ख}}{\alpha^4} + \frac{\beta^2\text{क}^4}{\alpha^6}\right) = \frac{3\gamma\text{क ख}^2}{\alpha^3} - \frac{6\beta\gamma\text{क}^3\text{ख}}{\alpha^5} + \frac{3\beta^2\text{क}^4}{\alpha^6}$$

Again

$$4\delta A^3B = 4\delta\frac{\text{क}^3}{\alpha^3}\left(\frac{\text{ख}}{\alpha} - \frac{\beta\text{क}^2}{\alpha^3}\right) = \frac{4\delta\text{क}^3\text{ख}}{\alpha^4} - \frac{4\beta\delta\text{क}^5}{\alpha^6}$$

And lastly

$$8A^5 = \frac{8\text{क}^5}{\alpha^5}$$

$$F = \begin{cases} \frac{\text{क}}{\alpha} - \frac{2\beta\text{क}\text{च}}{\alpha^3} - \frac{2\beta\text{ख}\text{ग}}{\alpha^4} - \frac{3\gamma\text{क}^2\text{ग}}{\alpha^4} - \frac{3\gamma\text{क}\text{ख}^2}{\alpha^4} \\ + \frac{6\beta^2\text{क}^2\text{ग}}{\alpha^5} + \frac{6\beta^2\text{क}\text{ख}^2}{\alpha^6} - \frac{4\delta\text{क}^5\text{ख}}{\alpha^6} + \frac{20\beta\gamma\text{क}^3\text{ख}}{\alpha^6} - \frac{\varepsilon\text{क}^5}{\alpha^6} \\ - \frac{20\beta^3\text{क}^3\text{ख}}{\alpha^7} + \frac{6\beta\delta\text{क}^5}{\alpha^7} + \frac{3\gamma^2\text{क}^5}{\alpha^7} - \frac{21\beta^2\gamma\text{क}^5}{\alpha^8} + \frac{14\beta^4\text{क}^5}{\alpha^9} \end{cases}$$

$$\left.\begin{array}{l} \alpha G + 2\beta AF + 2\beta BD + \beta C^2 + 3\gamma A^2 D + 6\gamma ABC \\ + \gamma B^3 + 4\delta A^3 C + 6\delta A^2 B^2 + 5\varepsilon A^4 B + \zeta A^6 \end{array}\right\} = \text{न}$$

$$\left.\begin{array}{l} \frac{2\beta\text{क}\text{छ}}{\alpha^2} - \frac{4\beta^2\text{क}^2\text{च}}{\alpha^4} - \frac{4\beta^2\text{क}\text{ख}\text{ग}}{\alpha^4} - \frac{6\beta\gamma\text{क}^3\text{ग}}{\alpha^5} - \frac{6\beta\gamma\text{क}^2\text{ख}^2}{\alpha^5} \\ + \frac{12\beta^3\text{क}^3\text{ग}}{\alpha^6} + \frac{12\beta^3\text{क}^2\text{ख}^2}{\alpha^6} - \frac{8\beta\delta\text{क}^4\text{ख}}{\alpha^6} + \frac{40\beta^2\gamma\text{क}^4\text{ख}}{\alpha^7} - \frac{2\beta\varepsilon\text{क}^6}{\alpha^7} \\ - \frac{40\beta^4\text{क}^4\text{ख}}{\alpha^8} + \frac{12\beta^2\delta\text{क}^6}{\alpha^8} + \frac{6\beta\gamma^2\text{क}^6}{\alpha^8} - \frac{42\beta^3\gamma\text{क}^6}{\alpha^9} + \frac{28\beta^5\text{क}^6}{\alpha^{10}} \end{array}\right\} = 2\beta AF$$

$$\frac{2\beta\text{ख}\text{च}}{\alpha^3} - \frac{4\beta^2\text{क}\text{ख}\text{ग}}{\alpha^4} - \frac{2\beta^2\text{ख}^3}{\alpha^4} - \frac{2\beta^2\text{क}^2\text{च}}{\alpha^4} - \frac{6\beta\gamma\text{क}^2\text{ख}^2}{\alpha^6} + \frac{14\beta^3\text{क}^2\text{ख}^2}{\alpha^6}$$

$$+ \frac{2\beta\delta\text{क}\text{ख}}{\alpha^6} + \frac{4\beta^2\text{क}^3\text{ग}}{\alpha^6} + \frac{16\beta^2\gamma\text{क}^4\text{ख}}{\alpha^7} - \frac{22\beta^4\text{क}^4\text{ख}}{\alpha^8} + \frac{2\beta^2\delta\text{क}^6}{\alpha^8}$$

$$- \frac{16\beta^3\gamma\text{क}^6}{\alpha^9} + \frac{16\beta^6\text{क}^6}{\alpha^{10}} = 2\beta BD$$

$$\beta C^2 = \begin{cases} \frac{\beta\text{ग}^2}{\alpha^2} - \frac{4\beta^2\text{क}\text{ख}\text{ग}}{\alpha^4} - \frac{2\beta\gamma\text{क}^3\text{ग}}{\alpha^4} + \frac{4\beta^3\text{क}^2\text{ख}^2}{\alpha^6} + \frac{4\beta^3\text{क}^3\text{ग}}{\alpha^6} \\ + \frac{4\beta^2\gamma\text{क}^4\text{ख}}{\alpha^7} + \frac{\beta\gamma^2\text{क}^6}{\alpha^8} - \frac{8\beta^4\text{क}^4\text{ख}}{\alpha^8} - \frac{4\beta^2\gamma\text{क}^6}{\alpha^9} + \frac{4\beta^6\text{क}^6}{\alpha^{10}} \end{cases}$$

$$3\gamma A^2 D = \begin{cases} \frac{3\gamma\text{क}^2\text{च}}{\alpha^5} - \frac{6\beta\gamma\text{क}^5\text{ग}}{\alpha^6} - \frac{3\beta\gamma\text{क}^4\text{ख}^2}{\alpha^6} - \frac{2\gamma^2\text{क}^4\text{ख}}{\alpha^6} \\ + \frac{18\beta^2\gamma\text{क}^4\text{ख}}{\alpha^7} - \frac{8\delta\gamma\text{क}^6}{\alpha^7} + \frac{18\beta\gamma^2\text{क}^6}{\alpha^8} - \frac{16\beta^3\gamma\text{क}^6}{\alpha^9} \end{cases}$$

$$6\gamma ABC = \begin{cases} \frac{6\gamma\text{क}\text{ख}\text{ग}}{\alpha^5} - \frac{12\beta\gamma\text{क}^4\text{ख}^2}{\alpha^6} - \frac{6\beta\gamma\text{क}^3\text{ग}}{\alpha^6} - \frac{6\gamma^2\text{क}^4\text{ख}}{\alpha^6} \\ + \frac{24\beta^2\gamma\text{क}^4\text{ख}}{\alpha^7} + \frac{6\beta\gamma^2\text{क}^6}{\alpha^8} - \frac{12\beta^3\gamma\text{क}^6}{\alpha^9} \end{cases}$$

$$\gamma B^4 = \frac{\gamma \kappa^4}{\alpha^4} - \frac{3\beta\gamma\kappa^3\chi}{\alpha^5} + \frac{3\beta^2\gamma\kappa^2\chi^2}{\alpha^6} - \frac{\beta^3\gamma\kappa^3}{\alpha^7}$$

$$4\delta A^3 C = \frac{4\delta\kappa^3\gamma}{\alpha^4} - \frac{3\beta\delta\kappa^2\chi}{\alpha^5} - \frac{4\gamma\delta\kappa^3}{\alpha^6} + \frac{3\beta^2\delta\kappa^2}{\alpha^7}$$

$$6\delta A^2 B^2 = \frac{6\delta\kappa^2\chi^2}{\alpha^4} - \frac{12\beta\delta\kappa^2\chi}{\alpha^5} + \frac{6\beta^2\delta\kappa^2}{\alpha^6}$$

$$4\varepsilon A B^3 = \frac{4\varepsilon\kappa\chi^3}{\alpha^4} - \frac{3\beta\varepsilon\kappa^3}{\alpha^5}$$

$$\zeta A^6 = \frac{\zeta\kappa^6}{\alpha^6}$$

$$G = \left| \begin{array}{l} \frac{\eta}{\alpha} - \frac{2\beta\kappa\theta}{\alpha^2} + \frac{2\beta\gamma\iota}{\alpha^3} - \frac{\beta\gamma^2}{\alpha^3} - \frac{6\gamma\kappa\chi\gamma}{\alpha^4} - \frac{\gamma\chi^3}{\alpha^4} - \frac{3\gamma\kappa^2\iota}{\alpha^4} \\ + \frac{4\beta^2\kappa\iota}{\alpha^5} + \frac{12\beta^2\kappa\chi\gamma}{\alpha^5} + \frac{2\beta^2\chi^3}{\alpha^5} + \frac{2\beta^2\kappa^2\iota}{\alpha^5} - \frac{4\delta\kappa^3\gamma}{\alpha^5} - \frac{6\delta\kappa^2\chi^2}{\alpha^6} \\ + \frac{20\beta\gamma\kappa^3\gamma}{\alpha^6} + \frac{30\beta\gamma\kappa^2\chi^2}{\alpha^6} - \frac{4\beta^3\kappa^3\chi^2}{\alpha^6} - \frac{6\varepsilon\kappa^4\chi}{\alpha^6} - \frac{16\beta^3\kappa^3\gamma}{\alpha^7} \\ - \frac{26\beta^3\kappa^2\chi^2}{\alpha^7} + \frac{30\beta\delta\kappa^4\chi}{\alpha^7} - \frac{4\beta^3\kappa^3\gamma}{\alpha^7} + \frac{15\gamma^2\kappa^4\chi}{\alpha^7} - \frac{\zeta\kappa^6}{\alpha^6} \\ - \frac{105\beta^3\gamma\kappa^4\chi}{\alpha^8} + \frac{7\beta\varepsilon\kappa^6}{\alpha^8} + \frac{7\delta\gamma\kappa^6}{\alpha^8} + \frac{70\beta^4\kappa^4\chi}{\alpha^8} - \frac{28\beta^5\gamma\kappa^6}{\alpha^8} \\ - \frac{28\beta\gamma^2\kappa^6}{\alpha^8} + \frac{83\beta^3\gamma\kappa^6}{\alpha^8} - \frac{44\beta^6\kappa^6}{\alpha^{10}} - \frac{4\beta^6\kappa^6}{\alpha^{10}} \end{array} \right|$$

First 6 differentials of y^6 computed.

$$d.y^6 = 6y^5.dy$$

$$d^2.y^6 = 30y^4.dy^2 + 6y^5.d^2y$$

$$d^3.y^6 = 120y^3.dy^3 + 90y^4.dy.d^2y + 6y^5.d^3y$$

$$d^4.y^6 = 360y^2.dy^4 + 720y^3.dy^2.d^2y + 90y^4.d^2y^2 + 120y^4.dy.d^3y + 6y^5.d^4y$$

$$d^5.y^6 = \begin{cases} 720y.dy^5 + 3600y^2.dy^3.d^2y + 1800y^3.dy.d^2y^2 + 1200y^3.dy^2.d^3y \\ + 300y^4.d^2y.d^3y + 150y^4.dy.d^4y + 6y^5.d^5y \end{cases}$$

$$d^6.y^6 = \begin{cases} 720dy^6 + 10800y.dy^4.d^2y + 16200y^2.dy^2.d^2y^2 + 7200y^2.dy^3.d^3y \\ + 1800y^3.d^2y^3 + 7200y^3.dy.d^2y.d^3y + 1800y^3.dy^2.d^4y \\ + 300y^4.d^3y^2 + 450y^4.d^2y.d^4y + 180y^4.dy.d^5y + 6y^5.d^6y \end{cases}$$

&c.

$$\frac{(4\cos 2B.\sin^3 B + 3\sin^2 2B.\sin B)(2\cos^4\alpha+\cos^2\alpha)}{\sin^4\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$-\frac{2\sin 2B.\sin^3 B.(10\cos^3\alpha+2\cos\alpha)}{\sin^5\alpha}\times\cos B.d\alpha$$

$$\frac{(4\cos 4B.\sin B+\sin 4B.\cos B)(8\cos^4\alpha+24\cos^2\alpha+1)}{4\sin^4\alpha}\times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$+\frac{\sin 4B.\sin B(80\cos^3\alpha+52\cos\alpha)}{4\sin^5\alpha}\times\cos B.d\alpha$$

$$-\frac{(2\sin 4B.\cos B-\sin^2 2B.\sin B)(2\cos^2\alpha+7)}{4\sin^4\alpha}\times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$+\frac{\sin^2 2B.\cos B.(40\cos^3\alpha+68\cos\alpha)}{4\sin^6\alpha}\times\cos B.d\alpha$$

$$=$$

$$\frac{d^6\alpha}{d\alpha^6} = -\frac{4\cos 2B.\sin^4 B(2\cos^4\alpha+\cos^2\alpha).\cos\alpha}{\sin^5\alpha} - \frac{3\sin^2 2B.\sin^2 B(2\cos^4\alpha+\cos^2\alpha)\cos\alpha}{\sin^5\alpha}$$

$$-\frac{\sin^2 2B.\sin^2 B.(10\cos^2\alpha+2\cos\alpha)}{\sin^6\alpha} + \frac{4\cos 4B.\sin^2 B(8\cos^4\alpha+24\cos^2\alpha+1).\cos\alpha}{4\sin^5\alpha}$$

$$+\frac{\sin 4B.\sin 2B.(8\cos^4\alpha+24\cos^2\alpha+1).\cos\alpha}{8\sin^5\alpha} + \frac{\sin 4B.\sin 2B.(80\cos^3\alpha+52\cos\alpha)}{8\sin^5\alpha}$$

$$+\frac{\sin 4B.\sin 2B.(20\cos^2\alpha+7).\cos\alpha}{4\sin^5\alpha} - \frac{\sin^2 2B.\sin^2 B.(20\cos^2\alpha+7)\cos\alpha}{4\sin^5\alpha}$$

$$+\frac{\sin^2 2B.\cos^2 B.(40\cos^3\alpha+68\cos\alpha)}{4\sin^5\alpha} =$$

$$-\frac{4\cos 2B.\sin^4 B.(2\cos^2\alpha+1).\cos^3\alpha}{\sin^6\alpha} - \frac{3\sin^2 2B\ \sin^2 B.(8\cos^4\alpha+24\cos^2\alpha+5)\cos\alpha}{4\sin^5\alpha}$$

$$+\frac{\cos 4B.\sin^2 B.(8\cos^4\alpha+24\cos^2\alpha+1)\cos\alpha}{\sin^6\alpha} + \frac{\sin 4B.\sin 2B.(8\cos^4\alpha+144\cos^2\alpha+67)\cos\alpha}{8\sin^5\alpha}$$

$$+\frac{\sin^2 2B.\cos^2 B.(40\cos^2\alpha+17).\cos\alpha}{\sin^6\alpha}$$

From 2nd to 6th differential of $\frac{dC}{da}$ Constant:

$$d.\frac{dC}{da} = -\frac{\cos B}{\sin a} \times dB = -\frac{\sin B.\cos a}{\sin^2 a} \times da =$$

$$\frac{\cos B}{\sin a} \times -\frac{\sin B.\cos a}{\sin a}.da = -\frac{\sin B.\cos a}{\sin^2 a}.\cos B.da$$

$$\frac{d^2C}{da^2} = -\frac{\sin 2B.\cos a}{\sin^2 a}$$

$$d.\frac{d^2C}{da^2} = -\frac{2\cos 2B.\cos a}{\sin^2 a} \times dB - \sin 2B.\left\{-\frac{\sin a}{\sin^2 a} - \frac{2\cos^2 a}{\sin^3 a}\right\} da =$$

$$-\frac{2\cos 2B.\cos a}{\sin^2 a} \times -\frac{\sin B.\cos a}{\sin^2 a}.da - \sin 2B. \times -\frac{(\cos^2 a+1)}{\sin^3 a}.\cos B.da$$

$$\frac{d^3C}{da^3} = \frac{2\cos 2B.\sin B.\cos^2 a}{\sin^3 a} + \frac{\sin 2B.\cos B(\cos^2 a+1)}{\sin^3 a}$$

$$d.\frac{d^3C}{da^3} = \frac{(-4\sin 2B.\sin B + 2\cos 2B.\cos B)\cos^2 a}{\sin^3 a} \times dB$$

$$+ 2\cos 2B.\sin B.\left(-\frac{2\cos a.\sin a}{\sin^3 a} - \frac{3\cos^2 a.\cos a}{\sin^4 a}\right).da$$

$$+ \frac{(2\cos 2B.\cos B - \sin 2B.\sin B)(\cos^2 a+1)}{\sin^3 a} \times dB$$

$$+ \sin 2B.\cos B.\left(\frac{-2\cos a.\sin a}{\sin^3 a} - \frac{3(\cos^2 a+1)\cos a}{\sin^4 a}\right).da =$$

$$\frac{(-4\sin 2B.\sin B + 2\cos 2B.\cos B)\cos^2\alpha}{\sin^3\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$-\frac{2\cos 2B.\sin B.(\cos^3\alpha + 2\cos\alpha)}{\sin^4\alpha} \times \cos B.d\alpha$$

$$+\frac{(2\cos 2B.\cos B - \sin 2B.\sin B)(\cos^2\alpha + 1)}{\sin^3\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$-\frac{\sin 2B.\cos B.(\cos^3\alpha + 5\cos\alpha)}{\sin^4\alpha} \times \cos B.d\alpha$$

$$\frac{d^4C}{d\alpha^4} = \frac{4\sin 2B.\sin^2 B.\cos^3\alpha}{\sin^4\alpha} - \frac{\frac{1}{2}\sin 4B.\cos^3\alpha}{\sin^4\alpha} - \frac{\frac{1}{2}\sin 4B(\cos^3\alpha + 2\cos\alpha)}{\sin^4\alpha}$$

$$-\frac{\frac{1}{2}\sin 4B.(\cos^2\alpha + 1)\cos\alpha}{\sin^4\alpha} + \frac{\sin 2B.\sin^2 B.(\cos^2\alpha + 1)\cos\alpha}{\sin^4\alpha} - \frac{\sin 2B.\cos^2 B.(\cos^3\alpha + 5\cos\alpha)}{\sin^4\alpha}$$

$$= \frac{\sin 2B.\sin^2 B.(5\cos^2\alpha + 1).\cos\alpha}{\sin^4\alpha} - \frac{3\sin 4B.(\cos^2\alpha + 1)\cos\alpha}{2\sin^4\alpha}$$

$$-\frac{\sin 2B.\cos^2 B.(\cos^2\alpha + 5)\cos\alpha}{\sin^4\alpha}$$

Ex.d.

$$-d.\frac{d^4C}{d\alpha^4} = \frac{(2\cos 2B.\sin^2 B + 2\sin 2B.\sin B.\cos B)(5\cos^3\alpha + \cos\alpha)}{\sin^4\alpha} \times dB$$

$$+\sin 2B.\sin^2 B.\left\{\frac{-15\cos^2\alpha.\sin\alpha - \sin\alpha}{\sin^4\alpha} - \frac{4(5\cos^3\alpha + \cos\alpha)\cos\alpha}{\sin^5\alpha}\right\} \times d\alpha$$

$$-\frac{12\cos 4B.(\cos^3\alpha + \cos\alpha)}{2\sin^4\alpha} \times dB - 3\sin 4B.\left\{\frac{-3\cos^2\alpha.\sin\alpha - \sin\alpha}{2\sin^4\alpha} - \frac{4(\cos^3\alpha + \cos\alpha)\cos\alpha}{2\sin^5\alpha}\right\} d\alpha$$

$$-\frac{(2\cos 2B.\cos^2 B - 2\sin 2B.\cos B.\sin B)(\cos^3\alpha + 5\cos\alpha)}{\sin^4\alpha} \times dB$$

$$-\sin 2B.\cos^2 B.\left\{\frac{-3\cos^2\alpha.\sin\alpha - 5\sin\alpha}{\sin^4\alpha} - \frac{4(\cos^3\alpha + 5\cos\alpha)\cos\alpha}{\sin^5\alpha}\right\} \times d\alpha =$$

$$\frac{(2\cos 2B.\sin^2 B + \sin^4 2B)(6\cos^2\alpha+\cos\alpha)}{\sin^4\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.dc$$

$$= \frac{\sin 2B.\sin^2 B\,(6\cos^4\alpha + 18\cos^2\alpha + 1)}{\sin^6\alpha} \times \cos B.dc$$

$$- \frac{12\cos 4B.(\cos^3\alpha+\cos\alpha)}{2\sin^4\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.dc$$

$$+ \frac{3\sin 4B.(\cos^4\alpha + 6\cos^2\alpha + 1)}{2\sin^6\alpha} \times \cos B.dc$$

$$= \frac{(2\cos 2B.\cos^2 B - \sin^2 2B)(\cos^3\alpha + 5\cos\alpha)}{\sin^4\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.dc$$

$$+ \frac{\sin 2B.\cos^2 B\,(\cos^4\alpha + 18\cos^2\alpha + 5)}{\sin^6\alpha} \times \cos B.dc$$

$$\frac{d^5C}{dc^5} = -\frac{2\cos 2B.\sin^3 B.(6\cos^3\alpha+\cos\alpha)\cos\alpha}{\sin^6\alpha} = \frac{\sin^2 2B.\sin B(6\cos^3\alpha+\cos\alpha)\cos\alpha}{\sin^6\alpha}$$

$$- \frac{\frac{1}{2}\sin^2 2B.\sin B.(5\cos^4\alpha + 18\cos^2\alpha + 1)}{\sin^6\alpha} + \frac{12\cos 4B.\sin B.(\cos^3\alpha+\cos\alpha)\cos\alpha}{2\sin^6\alpha}$$

$$+ \frac{3\sin 4B.\cos B.(\cos^4\alpha + 6\cos^2\alpha + 1)}{2\sin^6\alpha} + \frac{\frac{1}{2}\sin 4B.\cos B.(\cos^3\alpha + 5\cos\alpha)\cos\alpha}{\sin^6\alpha}$$

$$- \frac{\sin^2 2B.\sin B.(\cos^3\alpha + 5\cos\alpha)\cos\alpha}{\sin^6\alpha} + \frac{\sin 2B.\cos^2 B\,(\cos^4\alpha + 18\cos^2\alpha + 5)}{\sin^6\alpha}$$

$$= -\frac{2\cos 2B.\sin^3 B.(6\cos^2\alpha+1)\cos^2\alpha}{\sin^6\alpha}$$

$$- \frac{\sin^2 2B.\sin B.(17\cos^4\alpha + 30\cos^2\alpha + 1)}{2\sin^6\alpha}$$

$$+ \frac{6\cos 4B.\sin B.(\cos^2\alpha+1)\cos^2\alpha}{\sin^6\alpha}$$

$$+ \frac{\sin 4B.\cos B.(4\cos^4\alpha + 23\cos^2\alpha + 3)}{2\sin^6\alpha}$$

$$+ \frac{\sin 2B.\cos^2 B.(\cos^4\alpha + 18\cos^2\alpha + 5)}{\sin^6\alpha}$$

$$\frac{\alpha^4 B}{\alpha\alpha^4} = \frac{-2\sin 2B/\sin^2 B\,(\cos^3\alpha+\cos\alpha)\cos\alpha}{\sin^4\alpha} + \frac{\frac{1}{4}\sin 4B.(\cos^3\alpha+\cos\alpha)\cos\alpha}{\sin^4\alpha}$$

$$+\frac{\frac{1}{4}\sin 4B.(2\cos^2\alpha+1)}{\sin^4\alpha} + \frac{\sin 4B.\cos^2\alpha}{\sin^4\alpha} - \frac{2\sin 2B.\sin^2 B.\cos^2\alpha}{\sin^4\alpha}$$

$$+\frac{2\sin 2B.\cos^2 B.(2\cos^2\alpha+1)}{\sin^4\alpha} =$$

$$-\frac{2\sin 2B.\sin^2 B.(\cos^2\alpha+2)\cos^2\alpha}{\sin^4\alpha} + \frac{\sin 4B.(\cos^4\alpha+10\cos^2\alpha+1)}{4\sin^4\alpha}$$

$$+\frac{2\sin 2B.\cos^2 B.(2\cos^2\alpha+1)}{\sin^4\alpha}$$

$$\alpha.\frac{\alpha^4 B}{\alpha\alpha^4} = \left(-4\cos 2B.\sin^2 B - 4\sin 2B.\sin B.\cos B\right)\frac{(\cos^4\alpha+2\cos^2\alpha)}{\sin^4\alpha}.\alpha B$$

$$-2\sin 2B.\sin^2 B.\left\{\frac{-4\cos^3\alpha.\sin\alpha - 4\cos\alpha.\sin\alpha}{\sin^4\alpha} - \frac{4(\cos^4\alpha+2\cos^2\alpha).\cos\alpha}{\sin^5\alpha}\right\}\alpha\alpha$$

$$+4\cos 4B.\frac{(\cos^4\alpha+10\cos^2\alpha+1)}{4\sin^4\alpha}.\alpha B$$

$$+\sin 4B.\left\{\frac{-4\cos^3\alpha.\sin\alpha - 20\cos\alpha.\sin\alpha}{4\sin^4\alpha} - \frac{4(\cos^4\alpha+10\cos^2\alpha+1)\cos\alpha}{4\sin^5\alpha}\right\}\alpha\alpha$$

$$+\left(4\cos 2B.\cos^2 B - 4\sin 2B.\cos B.\sin B\right)\frac{(2\cos^2\alpha+1)}{\sin^4\alpha}.\alpha B$$

$$+2\sin 2B.\cos^2 B.\left\{\frac{-4\cos\alpha.\sin\alpha}{\sin^4\alpha} - \frac{4(2\cos^2\alpha+1)\cos\alpha}{\sin^5\alpha}\right\}\alpha\alpha =$$

$$\left(-4\cos 2B.\sin^2 B - 2\sin^2 2B\right)\frac{\left(\cos^4\alpha + 2\cos^2\alpha\right)}{\sin^4\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$+ 2\sin.2B.\sin^2 B.\frac{\left(8\cos^3\alpha + 4\cos\alpha\right)}{\sin^6\alpha}.\cos B.d\alpha$$

$$+ 4\cos 4B.\frac{\left(\cos^4\alpha + 10\cos^2\alpha + 1\right)}{4\sin^4\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$- \sin 4B.\frac{\left(24\cos^3\alpha + 24\cos\alpha\right)}{4\sin^5\alpha}.\cos B.d\alpha$$

$$+\left(4\cos 2B.\cos^2 B - 2\sin^2 2B\right)\frac{\left(2\cos^2\alpha + 1\right)}{\sin^4\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$- 2\sin 2B.\cos^2 B\,\frac{\left(4\cos^3\alpha + 8\cos\alpha\right)}{\sin^6\alpha}.\cos B.d\alpha$$

$$\frac{d^6 B}{d\alpha^6} = +\frac{4\cos 2B.\sin^3 B.\left(\cos^4\alpha + 2\cos^2\alpha\right)\cos\alpha}{\sin^6\alpha} + \frac{2\sin^2 2B.\sin B\left(\cos^3\alpha + \cos\right.}{\sin^6\alpha}$$

$$+\frac{\sin^2 2B.\sin B.\left(8\cos^3\alpha + 4\cos\alpha\right)}{\sin^6\alpha} - \frac{4\cos 4B.\sin B.\left(\cos^4\alpha + 10\cos^2\alpha + 1\right)\cos}{+\sin^6\alpha}$$

$$\frac{\sin 4B.\cos B.\left(24\cos^3\alpha + 24\cos\alpha\right)}{4\sin^6\alpha} - \frac{\sin 4B.\cos B.\left(2\cos^2\alpha + 1\right)}{\sin^6\alpha}$$

$$+\frac{2\sin^2 2B.\sin B.\left(2\cos^2\alpha + 1\right)\cos\alpha}{\sin^6\alpha} - \frac{2\sin 2B.\cos^3 B.\left(4\cos^3\alpha + 8\cos\right.}{\sin^6\alpha}$$

$$= \frac{4\cos 2B.\sin^3 B.\left(\cos^2\alpha + 2\right)\cos^3\alpha}{\sin^6\alpha}$$

$$+\frac{2\sin^2 2B.\sin B.\left(\cos^4\alpha + 8\cos^2\alpha + 3\right)\cos\alpha}{\sin^6\alpha}$$

$$-\frac{\cos 4B.\sin B.\left(\cos^4\alpha + 10\cos^2\alpha + 1\right)\cos\alpha}{\sin^6\alpha}$$

$$-\frac{\sin 4B.\cos B.\left(8\cos^3\alpha + 7\cos\alpha\right)}{\sin^6\alpha}$$

$$-\frac{8\sin 2B.\cos^3 B.\left(\cos^2\alpha + 2\right)\cos\alpha}{\sin^6\alpha}$$

$$\alpha.\frac{\alpha^5 B}{d\alpha^5} = \left(-8.\sin 2B.\sin^4 B + 12\cos 2B.\sin^2 B.\cos B\right)\frac{(\cos^6\alpha + 2\cos^3\alpha)}{\sin^5\alpha} \times dB$$

$$+ 4\cos 2B.\sin^3 B.\left\{\frac{-5\cos^4\alpha.\sin\alpha - 6\cos^2\alpha.\sin\alpha}{\sin^5\alpha} - \frac{5(\cos^5\alpha + 2\cos^3\alpha)\cos\alpha}{\sin^6\alpha}\right\}\times d\alpha$$

$$+\left(8\sin 2B.\cos 2B.\sin B + 2\sin^2 2B.\cos B\right)\frac{(\cos^5\alpha + 8\cos^3\alpha + 3\cos\alpha)}{\sin^5\alpha}.dB$$

$$+ 2\sin^2 2B.\sin B.\left\{\frac{-5\cos^4\alpha.\sin\alpha - 24\cos^2\alpha.\sin\alpha - 3\sin\alpha}{\sin^6\alpha} - \frac{5(\cos^5\alpha + 8\cos^3\alpha + 3\cos\alpha)\cos\alpha}{\sin^6\alpha}\right\}d\alpha$$

$$+\left(4\sin 4B.\sin B - \cos 4B.\cos B\right)\frac{(\cos^5\alpha + 10\cos^3\alpha + \cos\alpha)}{\sin^5\alpha}.dB$$

$$-\cos 4B.\sin B\left\{\frac{-5\cos^4\alpha.\sin\alpha - 80\cos^2\alpha.\sin\alpha - \sin\alpha}{\sin^6\alpha} - \frac{5(\cos^5\alpha + 10\cos^3\alpha + \cos\alpha)\cos\alpha}{\sin^6\alpha}\right\}d\alpha$$

$$-\left(4\cos 4B.\cos B - \sin 4B.\sin B\right)\frac{8\cos^3\alpha + 7\cos\alpha}{\sin^5\alpha}.dB$$

$$-\sin 4B.\cos B.\left\{\frac{-24\cos^2\alpha.\sin\alpha - 7\sin\alpha}{\sin^6\alpha} - \frac{5(8\cos^3\alpha + 7\cos\alpha)\cos\alpha}{\sin^6\alpha}\right\}d\alpha$$

$$-\left(16\cos 2B.\cos^3 B - 24\sin 2B.\cos^2 B.\sin B\right)\frac{(\cos^3\alpha + 2\cos\alpha)}{\sin^5\alpha}.dB$$

$$-8\sin 2B.\cos^3 B.\left\{\frac{-3\cos^2\alpha.\sin\alpha - 2\sin\alpha}{\sin^6\alpha} - \frac{5(\cos^3\alpha + 2\cos\alpha)\cos\alpha}{\sin^6\alpha}\right\}d\alpha =$$

$$\left(-8\sin 2B.\sin^3 B + 8\sin 4B.\sin B\right).\frac{(\cos^5\alpha + 2\cos^3\alpha)}{\sin^5\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$-4.\cos 2B.\sin^3 B\ \frac{(9\cos^4\alpha + 6\cos^2\alpha)}{\sin^6\alpha} \times \cos B.d\alpha$$

$$+\left(4\sin 4B.\sin B + 2\sin^2 2B.\cos B\right)\frac{(\cos^6\alpha + 8\cos^3\alpha + 3\cos\alpha)}{\sin^5\alpha} \times -\frac{\sin B\cos\alpha}{\sin\alpha}.d\alpha$$

$$-2\sin^2 2B.\sin B.\frac{(21\cos^4\alpha + 36\cos^2\alpha + 3)}{\sin^6\alpha} \times \cos B.d\alpha$$

$$+\left(4\sin 4B.\sin B - \cos 4B.\cos B\right)\frac{(\cos^6\alpha + 10\cos^3\alpha + \cos\alpha)}{\sin^6\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$+\cos 4B.\sin B.\frac{(25\cos^4\alpha + 34\cos^2\alpha + 1)}{\sin^6\alpha}.\cos B.d\alpha$$

$$-\left(4\cos 4B.\cos B - \sin 4B.\sin B\right).\frac{(8\cos^3\alpha + 7\cos\alpha)}{\sin^6\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$+\sin 4B.\cos B.\frac{(16\cos^4\alpha + 52\cos^2\alpha + 7)}{\sin^6\alpha} \times \cos B.d\alpha$$

$$-\left(16\cos 2B.\cos^3 B - 12\sin^2 2B.\cos B\right)\frac{(\cos^3\alpha + 2\cos\alpha)}{\sin^6\alpha} \times -\frac{\sin B.\cos\alpha}{\sin\alpha}.d\alpha$$

$$+8\sin 2B.\cos^3 B.\frac{(2\cos^4\alpha + 11\cos^2\alpha + 2)}{\sin^6\alpha}.\cos B.d\alpha$$

$$\frac{d^6 B}{d\alpha^6} = +\frac{8\sin 2B.\sin^4 B.(\cos^5\alpha + 2\cos^3\alpha).\cos\alpha}{\sin^6\alpha} - \frac{8\sin 4B.\sin^2 B(\cos^5\alpha + 2\cos^3\alpha)}{\sin^6\alpha}$$

$$-\frac{\sin 4B.\sin^2 B.(9\cos^4\alpha + 6\cos^2\alpha)}{\sin^6\alpha} - \frac{4\sin 4B.\sin^2 B.(\cos^6\alpha + 8\cos^3\alpha + 3\cos\alpha)\cos\alpha}{\sin^6\alpha}$$

$$-\frac{\sin^2 2B.(\cos^5\alpha + 8\cos^3\alpha + 3\cos\alpha)\cos\alpha}{\sin^6\alpha} - \frac{\sin^3 2B\,(21\cos^4\alpha + 36\cos^2\alpha + 3)}{\sin^6\alpha}$$

$$+\frac{4\sin 4B.\sin^2 B.(\cos^6\alpha + 10\cos^3\alpha + \cos\alpha)\cos\alpha}{\sin^6\alpha} + \frac{\frac{1}{2}\cos 4B.\sin 2B(\cos^5\alpha + 10\cos$$

$$+\frac{\frac{1}{2}\cos 4B.\sin 2B(25\cos^4\alpha + 34\cos^2\alpha + 1)}{\sin^6\alpha} + \frac{2\cos 4B.\sin 2B(8\cos^3\alpha + 7\cos\alpha)}{\sin^6\alpha}$$

$$- \frac{\sin 4B.\sin^2 B.(8\cos^3\alpha + 7\cos\alpha)\cos\alpha}{\sin^6\alpha} + \frac{\sin 4B.\cos^2 B.(16\cos^4\alpha + 8\cos^2\alpha + 7)}{\sin^6\alpha}$$

$$+ \frac{4\sin 4B.\cos^2 B.(\cos^3\alpha + 2\cos\alpha)\cos\alpha}{\sin^6\alpha} - \frac{6\sin^2 2B.(\cos^3\alpha + 2\cos\alpha)\cos\alpha}{\sin^6\alpha}$$

$$+ \frac{8\sin 2B.\cos^4 B.(2\cos^4\alpha + 11\cos^2\alpha + 2)}{\sin^6\alpha} =$$

$$+ \frac{8\sin 2B.\sin^4 B.(\cos^6\alpha + 2\cos^4\alpha)}{\sin^6\alpha}$$

$$- \frac{\sin 4B.\sin^2 B.(11\cos^6\alpha + 95\cos^4\alpha + 29\cos^2\alpha)}{\sin^6\alpha}$$

$$- \frac{\sin^2 2B(\cos^6\alpha + 35\cos^4\alpha + 51\cos^2\alpha + 3)}{\sin^6\alpha}$$

$$+ \frac{\cos 4B.\sin 2B.(\cos^6\alpha + 67\cos^4\alpha + 63\cos^2\alpha + 1)}{2\sin^6\alpha}$$

$$+ \frac{\sin 4B.\cos^2 B.(20\cos^4\alpha + 60\cos^2\alpha + 7)}{\sin^6\alpha}$$

$$+ \frac{8\sin 2B.\cos^4 B.(2\cos^4\alpha + 11\cos^2\alpha + 2)}{\sin^6\alpha}$$

Radius of Curvature.

Suppose it were required to determine to within a foot the radius of curvature for any Meridional Arc, the formula, to be used for this purpose may be investigated as follows:

$$\nu = a(1-e^2)\left(1-e^2\sin^2\lambda\right)^{-\frac{3}{2}}$$ which expanded becomes

$$a(1-e^2) + \frac{3a(1-e^2)e^2}{2}\cdot\sin^2\lambda + \frac{15a(1-e^2)e^4}{8}\cdot\sin^4\lambda + \frac{35a(1-e^2)e^6}{16}\cdot\sin^6\lambda$$

$$+ \frac{315a(1-e^2)\cdot e^8}{128}\cdot\sin^8\lambda + \frac{693a(1-e^2)e^{10}}{256}\cdot\sin^{10}\lambda + \frac{3003a(1-e^2)e^{12}}{1024}\cdot\sin^{12}\lambda$$

+ &c. &c. &c.

In this series only the first four terms are effective, the rest are small & may therefore be safely thrown out of consideration whence it follows that

$$\nu = m^0 + m'\sin^2\lambda + m''\sin^4\lambda + m'''\sin^6\lambda$$

in which $m^0 = a(1-e^2) =$ 20784048·60 Feet

$$m' = \frac{3a(1-e^2)\cdot e^2}{2} \qquad \text{L}(m') = 5.3158486$$

$$m'' = \frac{15a(1-e^2)e^4}{8} \qquad \text{L}(m'') = 3.2347858$$

$$m''' = \frac{35a(1-e^2)e^6}{16} \qquad \text{L}(m''') = 1.1237597$$

Example

$\lambda = 24^\circ$ $\quad$ L $\sin\lambda = \bar{1}.6093133$

L $\sin^2\lambda$	$\bar{1}.2186266$	L. $\sin^4\lambda$ ... $\bar{2}.4372532$
L (m')	3.3158486	L (m'') ... 3.2347858
	4.5344752	1.6720390
	84235.39	46.99
	46.99	
	0.06	
	207 84048.60	
	208 18331.04	Feet

L $\sin^6\lambda$	$\bar{3}.6558798$
L (m''') ...	1.1237597
	$\bar{2}.7796395$
	0.06

{ Radius of Curvature at the parallel of 24°

Latitude	Radii of Curvature	Difference	Latitude	Radii of Curvature	Difference	Latitude	Radii of Curvature	Difference
° ′	Feet		° ′	Feet		° ′	Feet	
8 0	207 88067.6		22 0	208 13122.6		35 0	208 52316.8	
		157.7			420.5			569.3
10	207 88225.2	3.3	10	208 13543.1	2.4	10	208 52886.1	1.3
		171.0			422.9			570.6
20	207 88396.2	3.4	20	208 13966.0	2.5	20	208 53456.7	1.2
		174.4			425.4			571.8
30	207 88570.6	3.3	30	208 14391.4	2.6	30	208 54028.5	1.2
		177.7			428.0			573.0
40	207 88748.3	3.4	40	208 14819.4	2.4	40	208 54601.5	1.1
		181.1			430.4			574.1
50	207 88929.4	3.4	50	208 15249.8	2.5	50	208 55175.6	1.1
9 0	207 89113.9	184.5	23 0	208 15682.7	432.9	36 0	208 55750.8	575.2

Required to compute the Number to the Axis Minor to the nearest foot.

$$V = a\left(1 - e^2 \sin^2\lambda\right)^{-\frac{1}{2}}$$ which developed becomes

$$a + \frac{ae^2}{2}\cdot\sin^2\lambda + \frac{3ae^4}{8}\sin^4\lambda + \frac{5ae^6}{16}\cdot\sin^6\lambda + \frac{35ae^8}{128}\sin^8\lambda$$

$$+ \frac{63ae^{10}}{256}\cdot\sin^{10}\lambda + \frac{231ae^{12}}{1024}\sin^{12}\lambda + \&c \quad \&c$$

In this series only the four terms are effective, & all the rest are

minute & superfluous. Hence

$$V = n^\circ + n'\sin^2\lambda + n''\sin^4\lambda + n'''\sin^6\lambda$$

in which $n^\circ = a$ 20922931.80 Feet

$$n' = \frac{ae^2}{2} \quad \text{L}(n') = 4.8416197$$

$$n'' = \frac{3ae^4}{8} \quad \text{L}(n'') = 2.5387082$$

$$n''' = \frac{5ae^6}{16} \quad \text{L}(n''') = 0.2815541$$

Example

$\lambda = 24^\circ$ ..... $L \sin\lambda = \bar{1}\cdot 6093133$

$L \sin^2\lambda = \bar{1}\cdot 2186266$ $\quad L \sin^4\lambda = \bar{2}\cdot 4372532$ $\quad L \sin^6\lambda = \bar{3}\cdot 6558798$

$L(n') = 4\cdot 8416197$ $\quad L(n'') \ldots 2\cdot 5387082$ $\quad L(n''') \ldots 0\cdot 2815541$

$4\cdot 0602463$ $\quad 0\cdot 9759614$ $\quad \bar{3}\cdot 9374339$

11488·05 $\quad$ 9·46 $\quad$ 0·01

9·46

0·01

20922931·80

20934429·32 Feet { Normal to the Axis Minor at the parallel of 24°

Latitude	Normal to the Axis Minor	Difference	Latitude	Normal to the Axis Minor	Difference	Latitude	Normal to the Axis Minor	Difference
° ′			° ′			° ′		
0	20924277·0		22 0	20932683·4		35 0	20945814·0	
		56·2			140·9			190·6
10	20924333·2	1·2	10	20932824·3	0·9	10	20946005·5	0·5
		57·4			141·8			191·1
20	20924390·6	1·1	20	20932966·1	0·8	20	20946196·6	0·3
		58·5			142·6			191·4
30	20924449·1	1·1	30	20933108·7	0·9	30	20946388·0	0·4
		59·6			143·5			191·8
40	20924508·7	1·2	40	20933252·2	0·8	40	20946579·8	0·4
		60·8			144·3			192·2
50	20924569·5	1·1	50	20933396·5	0·8	50	20946772·0	0·4
		61·9			145·1			192·6
0	20924631·4		23 0	20933541·6		36 0	20946964·6	

Factors	Multiples of cosec 1"	Factors
1	206 264" 806 247 096	1
2	412 529 612 494 192	2
3	618 794 418 741 288	3
4	825 059 224 988 384	4
5	1031 324 031 235 480	5
6	1237 588 837 482 576	6
7	1443 853 643 729 672	7
8	1650 118 449 976 768	8
9	1856 383 256 223 864	9

Exd by Sreenath Sicdar

Fundamental Term

$$\frac{180 \times 60 \times 60''}{3.141592653589793238 46} =$$

206264" 806247096 = cosec 1"

Log of Cosec 1" Computed

180° ... Log = 2. 25527 25051 03306 06980
3.6.00" Log = 3. 55630 25007 67287 26502
A.C of Log n = $\bar{1}$. 50285 01273 05866 14565
Log Cosec 1" = 5. 31442 51331 76459 48047

$$\frac{d\lambda}{ds} = \frac{(1-e^2\sin^2\lambda)^{\frac{3}{2}}}{a(1-e^2)}$$

$$\alpha\cdot\frac{d\lambda}{ds} = \frac{\frac{3}{2}\cdot(1-e^2\sin^2\lambda)^{\frac{1}{2}}\times -e^2\sin 2\lambda\cdot d\lambda}{a(1-e^2)}$$

$$\frac{d^2\lambda}{ds^2} = \frac{-\frac{3}{2}(1-e^2\sin^2\lambda)^2\cdot e^2\sin 2\lambda}{a^2(1-e^2)}$$

$$\alpha\cdot\frac{d^2\lambda}{ds^2} = \frac{-3(1-e^2\sin^2\lambda)\cdot e^2\sin 2\lambda\times -e^2\sin 2\lambda\cdot d\lambda}{a^2(1-e^2)^2}$$

$$\frac{-\frac{3}{2}\cdot(1-e^2\sin^2\lambda)^2\cdot e^2\times 2\cos 2\lambda\cdot d\lambda}{a^2(1-e^2)^2}$$

$$\frac{d^3\lambda}{ds^3} = +\frac{3(1-e^2\sin^2\lambda)^{\frac{5}{2}}\cdot e^4\cdot\sin^2 2\lambda}{a^3(1-e^2)^3} - \frac{3(1-e^2\sin^2\lambda)^{\frac{7}{2}}\cdot e^2\cdot\cos 2\lambda}{a^3(1-e^2)^3}$$

$$\alpha\cdot\frac{d^3\lambda}{ds^3} = \frac{+\frac{15}{2}\cdot(1-e^2\sin^2\lambda)^{\frac{3}{2}}\cdot e^4\cdot\sin^2 2\lambda\times -e^2\sin 2\lambda\cdot d\lambda}{a^3(1-e^2)^3}$$

$$+\frac{3(1-e^2\sin^2\lambda)^{\frac{5}{2}}\cdot e^4\times 2\sin 4\lambda\cdot d\lambda}{a^3(1-e^2)^3}$$

$$\frac{-\frac{21}{2}\cdot(1-e^2\sin^2\lambda)^{\frac{5}{2}}\cdot e^2\cdot\cos 2\lambda\times -2\sin 2\lambda\cdot d\lambda}{a^3(1-e^2)^3}$$

$$\frac{-3(1-e^2\sin^2\lambda)^{\frac{7}{2}}e^2\times -2\sin 2\lambda\cdot d\lambda}{a^3(1-e^2)^3}$$

$$\frac{d^4\lambda}{ds^4} = -\frac{\frac{15}{2}\cdot(1-e^2\sin^2\lambda)^3\cdot e^6\cdot\sin^3 2\lambda}{a^4(1-e^2)^4}$$

$$+\frac{\frac{45}{4}\cdot(1-e^2\sin^2\lambda)^4\cdot e^4\cdot\sin 4\lambda}{a^4(1-e^2)^4}$$

$$+\frac{6(1-e^2\sin^2\lambda)^5\cdot e^2\cdot\sin 2\lambda}{a^4(1-e^2)^4}$$

$$\alpha \cdot \frac{d^4\lambda}{\alpha s^4} = \frac{-\frac{45}{2}\cdot(1-e^2\cdot\sin^2\lambda)^2\cdot e^6\cdot\sin^2 2\lambda\,\alpha - e^2\cdot\sin 2\lambda\cdot\alpha\lambda}{a^4(1-e^2)^4} +$$

$$\frac{-\frac{15}{2}\cdot(1-e^2\cdot\sin^2\lambda)^3\cdot e^6\times 8\sin 2\lambda\cdot\sin 4\lambda\cdot\alpha\lambda}{a^4(1-e^2)^4}$$

$$+\frac{\frac{180}{4}\cdot(1-e^2\cdot\sin^2\lambda)^3\cdot e^4\cdot\sin 4\lambda\cdot\alpha - e^2\cdot\sin 2\lambda\cdot\alpha\lambda}{a^4(1-e^2)^4}$$

$$+\frac{\frac{45}{4}\cdot(1-e^2\cdot\sin^2\lambda)^4\cdot e^4\times 4\cos 4\lambda\cdot\alpha\lambda}{a^4\cdot(1-e^2)^4}$$

$$+\frac{30(1-e^2\sin^2\lambda)^4\cdot e^2\cdot\sin 2\lambda\,\alpha - e^2\cdot\sin 2\lambda\cdot\alpha\lambda}{a^4\cdot(1-e^2)^4}$$

$$+\frac{6(1-e^2\sin^2\lambda)^5\cdot e^2\times 2\cos 2\lambda\cdot\alpha\lambda}{a^4(1-e^2)^4}$$

$$\frac{\alpha^6\lambda}{\alpha s^6} = +\frac{\frac{45}{2}\cdot(1-e^2\cdot\sin^2\lambda)^{\frac{7}{2}}\cdot e^8\cdot\sin^4 2\lambda}{a^6(1-e^2)^6}$$

$$-\frac{\frac{135}{2}(1-e^2\sin^2\lambda)^{\frac{9}{2}}\cdot e^6\cdot\sin 4\lambda\cdot\sin 2\lambda}{a^6(1-e^2)^6}$$

$$+\frac{45(1-e^2\cdot\sin^2\lambda)^{\frac{11}{2}}\cdot e^4\cdot\cos 4\lambda}{a^6(1-e^2)^6}$$

$$-\frac{30(1-e^2\cdot\sin^2\lambda)^{\frac{11}{2}}\cdot e^4\cdot\sin^2 2\lambda}{a^6(1-e^2)^6}$$

$$+\frac{12(1-e^2\cdot\sin^2\lambda)^{\frac{13}{2}}\cdot e^2\cdot\cos 2\lambda}{a^6(1-e^2)^6}$$

Ex.

$$\alpha.\frac{\alpha^5\lambda}{d\sigma^6} = +\frac{\frac{315}{4}.(1-e^2.\sin^2\lambda)^{\frac{5}{2}}.e^8.\sin^4 2\lambda \times -e^2.\sin 2\lambda.\alpha\lambda}{a^5(1-e^2)^6}$$

$$+\frac{\frac{45}{2}.(1-e^2.\sin^2\lambda)^{\frac{7}{2}}.e^8.\times 4\sin^2 2\lambda.\sin 4\lambda.\alpha\lambda}{a^5(1-e^2)^6}$$

$$-\frac{\frac{1215}{4}.(1-e^2.\sin^2\lambda)^{\frac{7}{2}}.e^6.\sin 4\lambda.\sin 2\lambda \times -e^2.\sin 2\lambda.\alpha\lambda}{a^5(1-e^2)^6}$$

$$-\frac{\frac{135}{2}.(1-e^2.\sin^2\lambda)^{\frac{7}{2}}.e^6.\sin 2\lambda \times 4\cos 4\lambda.\alpha\lambda}{a^5(1-e^2)^6}$$

$$-\frac{\frac{135}{2}.(1-e^2.\sin^2\lambda)^{\frac{9}{2}}.e^6.\sin 4\lambda.\times 2\cos 2\lambda.\alpha\lambda}{a^6(1-e^2)^6}$$

$$+\frac{\frac{495}{2}.(1-e^2\sin^2\lambda)^{\frac{9}{2}}.e^4.\cos 4\lambda \times -e^2.\sin 2\lambda.\alpha\lambda}{a^5(1-e^2)^6}$$

$$+\frac{45\,(1-e^2.\sin^2\lambda)^{\frac{11}{2}}.e^4.\times -4\sin 4\lambda.\alpha\lambda}{a^5(1-e^2)^6}$$

$$-\frac{\frac{330}{2}.(1-e^2.\sin^2\lambda)^{\frac{9}{2}}.e^4.\sin^2 2\lambda \times -e^2.\sin 2\lambda.\alpha\lambda}{a^6(1-e^2)^6}$$

$$-\frac{30(1-e^2.\sin^2\lambda)^{\frac{11}{2}}.e^4\times 2\sin 4\lambda.\alpha\lambda}{a^6(1-e^2)^5}$$

$$+\frac{\frac{165}{2}(1-e^2.\sin^2\lambda)^{\frac{11}{2}}.e^2.\cos 2\lambda \times -e^2\sin 2\lambda.\alpha\lambda}{a^5(1-e^2)^5}$$

$$+\frac{12.(1-e^2.\sin^2\lambda)^{\frac{13}{2}}.e^2\times -2\sin 2\lambda.\alpha\lambda}{a^5(1-e^2)^5}$$

$$\frac{\alpha^5 \delta\lambda}{\alpha s^6} = -\frac{315}{4}\cdot\frac{(1-e^2\cdot\sin^2\lambda)^4\cdot e^{10}\cdot\sin^5 2\lambda}{a^5(1-e^2)^6}$$

$$+\frac{1575}{4}\cdot\frac{(1-e^2\cdot\sin^2\lambda)^5\cdot e^8\cdot\sin 4\lambda\cdot\sin^2 2\lambda}{a^5(1-e^2)^6}$$

$$-\frac{1035}{2}\cdot\frac{(1-e^2\cdot\sin^2\lambda)^6\cdot e^6\cdot\cos 4\lambda\cdot\sin 2\lambda}{a^5(1-e^2)^6}$$

$$-\frac{135\,(1-e^2\cdot\sin^2\lambda)^6\cdot e^6\cdot\sin 4\lambda\cdot\cos 2\lambda}{a^5(1-e^2)^6}$$

$$+\frac{165\,(1-e^2\cdot\sin^2\lambda)^6\cdot e^6\cdot\sin^3 2\lambda}{a^5(1-e^2)^6}$$

$$-\frac{279\cdot(1-e^2\sin^2\lambda)^7\cdot e^4\cdot\sin 4\lambda}{a^5(1-e^2)^6}$$

$$-\frac{24\cdot(1-e^2\cdot\sin^2\lambda)^8\cdot e^2\cdot\sin 2\lambda}{a^5(1-e^2)^6}$$

given the length of a meridional arc in feet or in any other linear measure, it is required to convert it to seconds.

Let S be an arc of the meridian extending from the Equator to the parallel of λ, it follows that

$$\lambda = fS$$

Now suppose S to vary and become $S + \Delta S$, this will produce a corresponding change in λ. Designating the altered value of λ by $\lambda +$ there will result

$$\lambda + \Delta\lambda = f(S + \Delta S)$$

If this expression is expanded by Taylor's Theorem, it will take the following form

$$\lambda + \Delta\lambda = \lambda + \frac{d\lambda}{dS}\cdot\frac{\Delta S}{1} + \frac{d^2\lambda}{dS^2}\cdot\frac{\Delta S^2}{1\cdot 2} + \frac{d^3\lambda}{dS^3}\cdot\frac{\Delta S^3}{1\cdot 2\cdot 3} + \ldots\ldots \quad (1)$$

Now substitute $-\Delta S$ for $+\Delta S$ & there will result

$$\lambda - \Delta\lambda' = \lambda - \frac{d\lambda}{dS}\cdot\frac{\Delta S}{1} + \frac{d^2\lambda}{dS^2}\cdot\frac{\Delta S^2}{1\cdot 2} - \frac{d^3\lambda}{dS^3}\cdot\frac{\Delta S^3}{1\cdot 2\cdot 3} + \ldots\ldots \quad (2)$$

Development (1) will convert ΔS into arc when the Latitude of its southern extremity is known. Similar conversion will be effected by Expression (2) when the Latitude of the northern limit of the arc $\overset{(\Delta S.)}{\wedge}$ is given.

It sometimes happens in practice that the mean Latitude of ΔS is given.. In this case ΔS may be supposed to consist of two equal parts one lying on the northern and the other on the southern side of the Middle Point.. If therefore $\frac{1}{2}\Delta S$ is substituted for ΔS in Forms 1 & 2 the resulting expressions will be the values in arc of the two parts into which ΔS is divided.. Thus

$$\lambda + \Delta\lambda_{(1)} = \lambda + \frac{d\lambda}{dS}\cdot\frac{\Delta S}{2} + \frac{d^2\lambda}{dS^2}\cdot\frac{\Delta S^2}{2^2\cdot 2} + \frac{d^3\lambda}{dS^3}\cdot\frac{\Delta S^3}{2^3\cdot 2\cdot 3} + \ldots\ldots$$

$$\lambda - \Delta\lambda'_{(1)} = \lambda - \frac{d\lambda}{dS}\cdot\frac{\Delta S}{2} + \frac{d^2\lambda}{dS^2}\cdot\frac{\Delta S^2}{2^2\cdot 2} - \frac{d^3\lambda}{dS^3}\cdot\frac{\Delta S^3}{2^3\cdot 2\cdot 3} + \ldots\ldots\ldots$$

Now subtract the latter of these expressions from the former, the difference will give the value of ΔS in seconds..

$$\Delta\lambda_{(1)} + \Delta\lambda'_{(1)} = \frac{d\lambda}{dS}\cdot\Delta S + \frac{d^3\lambda}{dS^3}\cdot\frac{\overset{S}{\Delta\lambda^3}}{2^2\cdot 2\cdot 3} + \frac{d^5\lambda}{dS^5}\cdot\frac{\Delta S^5}{2^4\cdot 2\cdot 3\cdot 4\cdot 5} + \ldots\ldots\ldots (3)$$

To determine the values of the differential coefficients it will be remarked that $\nu\cdot d\lambda = dS$.. ν standing for the radius of curvature for the meridional Section of the Terrestrial Spheroid.. Hence

$\frac{d\lambda}{dS} = \frac{1}{\nu}$, By successive differentiation of this form it will be seen that $\frac{d^2\lambda}{dS^2} = -\frac{3e^2\cdot\sin 2\lambda}{2(1-e^2)\cdot V\cdot\nu}$

$$\frac{d^3\lambda}{dS^3} = -\frac{3e^2\cdot\cos 2\lambda}{(1-e^2)\cdot V\cdot\nu^2} + \frac{3e^4\cdot\sin^2 2\lambda}{(1-e^2)^2\cdot V^2\cdot\nu}$$

$$\frac{d^4\lambda}{ds^4} = + \frac{6\, e^2 . \sin 2\lambda}{(1-e^2)\, V . N^3} + \frac{45\, e^4 . \sin 4\lambda}{4(1-e^2)^2 . V^2 . N^2} - \frac{15\, e^6 . \sin^3 2\lambda}{2(1-e^2)^3 . V^3 . N}$$

$$\frac{d^5\lambda}{ds^5} = \begin{cases} + \dfrac{12\, e^2 . \cos 2\lambda}{(1-e^2)\, V . N^4} - \dfrac{30\, e^4 . \sin^2 2\lambda}{(1-e^2)^2 . V^2 . N^3} + \dfrac{45\, e^4 . \cos 4\lambda}{(1-e^2)^2 . V^2 . N^3} \\ - \dfrac{135 . e^6 . \sin 4\lambda . \sin 2\lambda}{2(1-e^2)^3 . V^3 . N^2} + \dfrac{45\, e^8 . \sin^4 2\lambda}{2(1-e^2)^4 . V^4 . N} \end{cases}$$

$$\frac{d^6\lambda}{ds^6} = \begin{cases} - \dfrac{24\, e^2 . \sin 2\lambda}{(1-e^2) . V . N^5} - \dfrac{279\, e^4 . \sin 4\lambda}{(1-e^2)^2 . V^2 . N^4} + \dfrac{165 . e^6 . \sin^3 2\lambda}{(1-e^2)^3 . V^3 . N^3} - \dfrac{135\, e \ldots}{(1-e \ldots} \\ - \dfrac{1035\, e^6 . \cos 4\lambda . \sin 2\lambda}{(1-e^2)^3 . V^3 . N^3} + \dfrac{1575\, e^8 . \sin 4\lambda . \sin^2 \lambda}{4(1-e^2)^4 . V^4 . N^2} - \dfrac{315\, e^{10} \ldots}{4(1-e \ldots} \end{cases}$$

These values of the Differential coefficients being subs[tituted] in Developments 1, 2, & 3, there into arise the following ex[pressions] for the conversion of ΔS into seconds.

1st. When λ = Latitude of the Southern extremity of ΔS

$$\Delta\lambda = \begin{cases} \dfrac{\Delta S}{N} - \dfrac{3 e^2 . \sin 2\lambda . \Delta S^2}{4(1-e^2)\, V . N} - \dfrac{e^2 . \cos 2\lambda . \Delta S^3}{2(1-e^2) . V . N^2} + \dfrac{e^4 . \sin^2 2\lambda . \Delta S^3}{2(1-e^2)^2 . V^2 . N} \\ + \dfrac{e^2 . \sin 2\lambda . \Delta S^4}{4(1-e^2)\, V . N^3} + \dfrac{15\, e^4 . \sin 4\lambda . \Delta S^4}{32(1-e^2)^2 . V^2 . N^2} - \dfrac{5\, e^6 . \sin^3 2\lambda . \Delta S^4}{16(1-e^2)^3 . V^3 . N} \\ + \dfrac{e^2 . \cos 2\lambda . \Delta S^5}{10(1-e^2) . V . N^4} - \dfrac{e^4 . \sin^2 2\lambda . \Delta S^5}{4(1-e^2)^2 . V^2 . N^3} + \dfrac{3\, e^4 . \cos 4\lambda . \Delta S^5}{8(1-e^2)^2 . V^2 . N^3} \\ - \dfrac{9\, e^6 . \sin 4\lambda . \sin 2\lambda . \Delta S^5}{16(1-e^2)^3 . V^3 . N^2} + \dfrac{3\, e^8 . \sin^4 2\lambda . \Delta S^5}{16(1-e^2)^4 . V^4 . N} - \dfrac{e^2 . \sin 2\lambda \ldots}{30(1-e^2) \ldots} \\ - \dfrac{31\, e^4 . \sin 4\lambda . \Delta S^6}{80(1-e^2)^2 . V^2 . N^4} + \dfrac{11\, e^6 . \sin^3 2\lambda . \Delta S^6}{48(1-e^2)^3 . V^3 . N^3} - \dfrac{3\, e^6 . \sin 4\lambda . \cos \ldots}{16(1-e^2)^3 . V \ldots} \\ - \dfrac{23\, e^6 . \cos 4\lambda . \sin 2\lambda . \Delta S^6}{16 . (1-e^2)^3 . V^3 . N^3} + \dfrac{35\, e^8 . \sin 4\lambda . \sin^2 2\lambda . \Delta S^6}{64(1-e^2)^4 . V^4 . N^2} - \dfrac{7 e^{10} . \sin \ldots}{64(1- \ldots} \end{cases}$$

e^{iv} When λ = Latitude of the northern extremity of ΔS

$$= \left\{ \begin{array}{l} \frac{\Delta S}{N} + \frac{3e^2.\sin 2\lambda.\Delta S^2}{4(1-e^2)V.N} - \frac{e^2.\cos 2\lambda.\Delta S^3}{2(1-e^2)V.N^2} + \frac{e^4.\sin^2 2\lambda.\Delta S^3}{2(1-e^2)^2.V^2.N} \\ - \frac{e^2.\sin 2\lambda.\Delta S^4}{4(1-e^2)V.N^3} - \frac{15e^4.\sin 4\lambda.\Delta S^4}{32(1-e^2)^2.V^2.N^2} + \frac{5e^4.\sin^2 2\lambda.\Delta S^4}{16(1-e^2)^2.V^2.N^2} \\ + \frac{e^2.\cos 2\lambda.\Delta S^5}{10(1-e^2)V.N^4} - \frac{e^4.\sin^2 2\lambda.\Delta S^5}{4(1-e^2)^2.V^2.N^3} + \frac{3e^4.\cos 4\lambda.\Delta S^5}{8(1-e^2)^2.V^2.N^3} \\ - \frac{9e^6.\sin 4\lambda.\sin 2\lambda.\Delta S^5}{16(1-e^2)^3.V^3.N^2} + \frac{3e^8.\sin^4 2\lambda.\Delta S^6}{16(1-e^2)^4.V^4.N} + \frac{e^6.\sin 2\lambda.\Delta S^6}{30(1-e^2).V.N^4} \\ + \frac{21e^4.\sin 4\lambda.\Delta S^6}{80(1-e^2)^2.V^2.N^4} - \frac{11e^6.\sin^2 2\lambda.\Delta S^6}{48(1-e^2)^3.V^3.N^3} + \frac{3e^6.\sin 4\lambda.\cos 2\lambda.\Delta S^6}{16(1-e^2)^3.V^3.N^3} \\ + \frac{23e^6.\cos 4\lambda.\sin 2\lambda.\Delta S^6}{16(1-e^2)^3.V^3.N^3} - \frac{35e^8.\sin 4\lambda.\sin^2 2\lambda.\Delta S^6}{64.(1-e^2)^4.V^4.N^2} + \frac{7e^8.\sin^6 2\lambda.\Delta S^6}{64(1-e^2)^6.V^6.N} \end{array} \right.$$

2nd When λ = Mean Latitude of ΔS

$$+\Delta\lambda'' = \left\{ \begin{array}{l} \frac{\Delta S}{N} - \frac{e^2.\cos 2\lambda.\Delta S^3}{8(1-e^2).V.N^2} + \frac{e^4.\sin^2 2\lambda.\Delta S^3}{8(1-e^2)^2.V^2.N} \\ + \frac{e^2.\cos 2\lambda.\Delta S^6}{160(1-e^2).V.N^4} - \frac{e^4.\sin^2 2\lambda.\Delta S^6}{64(1-e^2)^2.V^2.N^3} + \frac{3e^4.\cos 4\lambda.\Delta S^6}{128(1-e^2)^2.V^2.N^3} \\ - \frac{9e^6.\sin 4\lambda.\sin 2\lambda.\Delta S^6}{256(1-e^2)^3.V^3.N^2} + \frac{3e^8.\sin^4 2\lambda.\Delta S^8}{256(1-e^2)^4.V^4.N} \end{array} \right.$$

Since the resulting values in these series

series will be required in seconds, it is evident that that result will be obtained by multiplying each term on the right hand side of the Equation by cosec 1".

In the application of these series to cases where the given Meridional Arc is large & in which great acc is required, all that will be necessary is to derive the 1st term $\frac{\Delta S}{N}$ cosec 1" from an arithmetical computation. The same pro procedure need not be followed with respect to the remainder of the terms which are small and which would entail no error if deduced Logarithmically.

But in the arithmetical deduction of 1st term, it will be necessary to determine N to within foot. The formula & computation by which this may effected have been given at p. p.

Similarly $\delta b = \frac{c \cdot \sin B}{\sin C} \cdot (\cot B \cdot \delta B - \cot C \cdot \delta C)$

By putting $\delta a = 0$ & $\delta b = 0$ we have

$\cot A \cdot \delta A - \cot C \cdot \delta C = 0$ ..... $\delta A = \frac{\cot C}{\cot A} \cdot \delta C$

& $\cot B \cdot \delta B + \cot C \cdot \delta C = 0$ ..... $\delta B = \frac{\cot C}{\cot B} \cdot \delta C$

Hence $\delta A + \delta B + \delta C = \left(\frac{\cot C}{\cot A} + \frac{\cot C}{\cot B} + 1 \right) \delta C = \varepsilon$

Wherefore $\delta C = \frac{\cot A \cdot \cot B \cdot \varepsilon}{\cot A \cdot \cot B + \cot A \cdot \cot C + \cot B \cdot \cot C} = \frac{\tan C \cdot \varepsilon}{\tan A + \tan B + \tan C}$

Similarly $\delta B = \frac{\tan B \cdot \varepsilon}{\tan A + \tan B + \tan C}$ & $\delta A = \frac{\tan A \cdot \varepsilon}{\tan A + \tan B + \tan C}$

To adapt these expressions to Logarithmic computation it may be remarked that $A + B + C = \pi + \varepsilon$, & since ε is small, $\tan A + \tan B + \tan C$ may be safely assumed $= \tan A \cdot \tan B \cdot \tan C$.. observing this substitution there will arise

$\delta A = \frac{\varepsilon}{\tan B \cdot \tan C}$ $\delta B = \frac{\varepsilon}{\tan A \cdot \tan C}$ $\delta C = \frac{\varepsilon}{\tan A \cdot \tan B}$

Which are the values of the required corrections.

Example No. 1

Base = unity

	Observed ∠s		sines	Logs.
A =	75° 2' 30"	.....	9.9850283	
B =	60 0 40	ε = 20"	9.9375792	b = 0.0884524
C =	44 57 10		9.8491268	a = 0.1359075

	Obs^d. ∠s	Corr^n.	Corrected ∠s	sines	
A =	75° 2' 30"	−11.56	75 2 18.44	9.9850218	
B =	60 0 40	−5.35	60 0 34.65	9.9375728	b = 0.088[illegible]
C =	44 57 10	−3.09	44 57 6.91	9.8491202	a = 0.135[illegible]

Example No. 2

	Obs^d. ∠s		sines	Logs.
A =	120° 2' 50"		9.9373238	
B =	39 57 10	ε = 20"	9.8076406	b = 0.2734732
C =	20 0 20		9.5341674	a = 0.4031564

	Obs^d. ∠s	Corr^n.	Corrected ∠s	sines.	
A =	120° 2' 50"	−65.58"	120° 1' 44.42"	9.9374036	
B =	39 57 10	+31.77	39 57 41.77	9.8077204	b = 0.27[illegible]
C =	20 0 20	+13.81	20 0 33.81	9.5342472	a = 0.40[illegible]

When an angle of the given Triangle exceeds ½π, the corrections appertaining thereto should have a sign contrary to that possessed by the corrections of the other two angles.

On Interpolation

Suppose m, to be any number whose logarithm is required, m' is not exactly found in the Table. Let the Tabular number next less be denoted by m', and its logarithm by q. Then calling p, the required logarithm of m, it will be found that

$$p = q + \frac{x}{1}\cdot d' + \frac{x\cdot\overline{x-1}}{1\cdot 2}\cdot d'' + \frac{x\cdot\overline{x-1}\cdot\overline{x-2}}{1\cdot 2\cdot 3}\cdot d''' + \frac{x\cdot\overline{x-1}\cdot\overline{x-2}\cdot\overline{x-3}}{1\cdot 2\cdot 3\cdot 4}\cdot d^{IV} + \frac{x\cdot\overline{x-1}\cdot\overline{x-2}\cdot\overline{x-3}\cdot\overline{x-4}}{1\cdot 2\cdot 3\cdot 4\cdot 5}\cdot d^{V} + \frac{x\cdot\overline{x-1}\cdot\overline{x-2}\cdot\overline{x-3}\cdot\overline{x-4}\cdot\overline{x-5}}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}\cdot d^{VI} + \ldots\ldots \quad (1)$$

in which x is the excess of m above m' and d', d'', d''', d^{IV}, d^{V}, d^{VI} are differences from the first to the Sixth order taken in succession.

Let Series (1) be arranged according to the powers of x, and it will take the following form.

$$\begin{aligned} p = q &+ x\cdot\{d' - \tfrac{1}{2}d'' + \tfrac{1}{3}d''' - \tfrac{1}{4}d^{IV} + \tfrac{1}{5}d^{V} - \tfrac{1}{6}d^{VI} + \ldots\ldots\} \\ &+ x^2\cdot\{\tfrac{1}{2}d'' - \tfrac{1}{2}d''' + \tfrac{11}{24}d^{IV} - \tfrac{5}{12}d^{V} + \tfrac{137}{360}d^{VI} - \ldots\ldots\} \\ &+ x^3\cdot\{\tfrac{1}{6}d''' - \tfrac{1}{4}d^{IV} + \tfrac{7}{24}d^{V} - \tfrac{5}{16}d^{VI} + \ldots\ldots\} \\ &+ x^4\cdot\{\tfrac{1}{24}d^{IV} - \tfrac{1}{12}d^{V} + \tfrac{17}{144}d^{VI} - \ldots\ldots\} \\ &+ x^5\cdot\{\tfrac{1}{120}d^{V} - \tfrac{1}{43}d^{VI} + \ldots\ldots\} \\ &+ x^6\cdot\{\tfrac{1}{720}d^{VI} - \ldots\ldots\} \end{aligned} \quad (2)$$

In practice Series (2) is susceptible of easier computation than Series (1). the first term being computed arithmetically and the remaining terms logarithmically.

Example

Required the Logarithm of 109625, 1488002

Tabular Numbers	Logarithms	Differences +	Differences −
10962	039 8897 977		
		396164	36
10963	039 9294 141		
		396128	
10964	039 9690 269		37
		396091	
10965	040 0086 360		36
		396055	
10966	040 0482 415		

Hence $q = 039\ 8897\ 977$, $d' = +\,396\ 164$, $d'' = -36$

$\frac{1}{2}d'' = -18$, $d' - \frac{1}{2}d'' = 396\ 182$,

$x = \left\{ \begin{matrix} 1096\,2514\,88002 \\ -1096\,2 \end{matrix} \right\} = \cdot 51488002$

The reason of the decimal point being placed before 5 is this: The figure in the natural numbers reckoning from left is obviously the place of Tabular unit. This arrangement being adhered to, it will follow that 5 is the Tenth's place, 1 the Hundredth's place &c. &c.

~~This view of the matter will have no effect upon the decimal~~ part (which is the result sought) of the required Logarithm, because that depends upon the figures of which a given number consists, and not on the position of the decimal point.

$q = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 039\ 8897\ 977$

$x.(d' - \frac{1}{2}d'') = \left\{ \begin{matrix}\text{obtained by} \\ \text{multiplication}\end{matrix} \right\} \ldots\ldots\ldots\ +\ 203\ 986{\cdot}20$

$+\,x^2 \times \frac{1}{2}d'' = \left\{ \begin{matrix}\text{derived from a} \\ \text{logarithmic computation}\end{matrix} \right\} \ldots\ldots\ -4{\cdot}77$

Required Logarithm of the given number .... } 039 9101 958

The foregoing computation is likewise applicable to the case in the Arc being given; its sine or cosine, or tangent or cotangent is re[quired].

The problem solved above is one case of interpolation, the case is this: Given a Logarithm, to compute its natural number.

In this case p becomes the given logarithm, q the Tabular logarithm next less; the natural number corresponding thereto being d', d''; d''', d^{iv}, d^{v}, d^{vi}, stand as before for the several orders of the differences. Now all these are known terms, the unknown term to be determined being x.

To find the value of x, take series (2), and transpose q to the left side of the equation. Then divide $p-q$ and the coefficients of $x^2, x^3, x^4, \ldots\ldots$ by the coefficient of x, after which substitute as follows for the quotients.

$$y = \frac{p-q}{d - \frac{1}{2}d' + \frac{1}{3}d'' - \frac{1}{4}d''' + \frac{1}{5}d^{iv} - \frac{1}{6}d^{v} + \ldots\ldots}$$

$$A = \frac{\frac{1}{2}d' - \frac{1}{2}d'' + \frac{11}{24}d''' - \frac{5}{12}d^{iv} + \frac{137}{360}d^{v} + \ldots\ldots}{d - \frac{1}{2}d' + \frac{1}{3}d'' - \frac{1}{4}d''' + \frac{1}{5}d^{iv} - \frac{1}{6}d^{v} + \ldots\ldots}$$

$$B = \frac{\frac{1}{6}d'' - \frac{1}{4}d''' + \frac{7}{24}d^{iv} - \frac{5}{16}d^{v} + \ldots\ldots}{d - \frac{1}{2}d' + \frac{1}{3}d'' - \frac{1}{4}d''' + \frac{1}{5}d^{iv} - \frac{1}{6}d^{v} + \ldots\ldots}$$

$$C = \frac{\frac{1}{24}\cdot d'' - \frac{1}{12}\cdot d^{iv} + \frac{17}{144}\cdot d^{v} - \ldots\ldots}{d - \frac{1}{2}\cdot d' + \frac{1}{3}\cdot d'' - \frac{1}{4}\cdot d''' + \frac{1}{5}\cdot d^{iv} - \frac{1}{6}\cdot d^{v} + \ldots\ldots}$$

$$D = \frac{\frac{1}{20}\cdot d^{iv} - \frac{1}{48}\cdot d^{v} + \ldots\ldots}{d - \frac{1}{2}\cdot d' + \frac{1}{3}\cdot d'' - \frac{1}{4}\cdot d''' + \frac{1}{5}\cdot d^{iv} - \frac{1}{6}\cdot d^{v} + \ldots\ldots}$$

$$E = \frac{\frac{1}{120}\cdot d^{v} - \ldots\ldots}{d - \frac{1}{2}\cdot d' + \frac{1}{3}\cdot d'' - \frac{1}{4}\cdot d''' + \frac{1}{5}\cdot d^{iv} - \frac{1}{6}\cdot d^{v} + \ldots\ldots}$$

Where

$$y = x + Ax^2 + Bx^3 + Cx^4 + Dx^5 + Ex^6 + \ldots\ldots$$

Now assume $x = y + \alpha y^2 + \beta y^3 + \gamma y^4 + \delta y^5 + \varepsilon y^6 + \ldots\ldots$

in which $\alpha.\beta.\gamma.\delta.\varepsilon\ldots\ldots$ may be determined by the method of indeterminate coefficients in the following manner.

$$x = y + \alpha y^2 + \beta y^3 + \gamma y^4 + \delta y^5 + \varepsilon y^6 \ldots\ldots$$

$$\left.\begin{array}{lllll}
A x^2 = & + A y^2 & + 2A\alpha y^3 & + 2A\beta \\ & & & + A\alpha^2 \end{array}\right\} y^4 \quad \left.\begin{array}{l} + 2A\gamma \\ + 2A\alpha\beta \end{array}\right\} y^5 \quad \left.\begin{array}{l} + 2A\delta \\ + 2A\alpha\gamma \\ + A\beta^2 \end{array}\right\} y^6 \ldots\ldots$$

$$\left.\begin{array}{lllll}
B x^3 = & & + B y^3 & + 3B\alpha y^4 & + 3B\beta \\ & & & & + 3B\alpha^2 \end{array}\right\} y^5 \quad \left.\begin{array}{l} + 3B\gamma \\ + 6B\alpha\beta \\ + B\alpha^3 \end{array}\right\} y^6 \ldots\ldots$$

$$\left.\begin{array}{lllll}
C x^4 = & & & + C y^4 & + 4C\alpha \cdot y^5 \end{array}\right. \quad \left.\begin{array}{l} + 4C\beta \\ + 6C\alpha^2 \end{array}\right\} y^6 \ldots\ldots$$

$$\left.\begin{array}{lllll}
D x^5 = & & & & + D y^5 \end{array}\right. \quad \left.\begin{array}{l} + 5D\alpha \cdot y^6 \ldots\ldots \end{array}\right.$$

$$\left.\begin{array}{lllll}
E x^6 = & & & & \end{array}\right. \quad \left.\begin{array}{l} + E y^6 \ldots\ldots \end{array}\right.$$

$$= y$$

Now equating the coefficients of the like powers of y, there will arise the following equations.

1st ..... $A + \alpha = 0$

2nd ..... $\beta + 2A\alpha + B = 0$

3rd ..... $\gamma + 2A\beta + A\alpha^2 \cancel{+ 3B\alpha + C = 0}$

4th ..... $\delta + 2A\gamma + 2A\alpha\beta + 3B\beta + 3B\alpha^2 + 4C\alpha + D = 0$

5th ..... $\varepsilon + 2A\delta + \cancel{2A\alpha\gamma + A\beta^2 + 3B\gamma} \overset{(+6B\alpha\beta)}{\wedge} \cancel{+ B\alpha + 4C\beta + 6C\alpha^2} + 5D\alpha + E = 0$

Which solved will give

1st ..... $\alpha = -A$

2nd ..... $\beta = 2A^2 - B$

3rd ..... $\gamma = -5A^3 + 5AB - C$

4th ..... $\delta = 14A^4 - 21A^2B + 6AC + 3B^2 - D$

5th ..... $\varepsilon = -42A^5 + 84A^3B - 28A^2C - 28AB^2 + 7BC + 7AD - E$

Hence

$$x = y - Ay^2 \left.\begin{matrix} + 2A^2 \\ - B \end{matrix}\right\} y^3 \left.\begin{matrix} - 5A^3 \\ + 5AB \\ - C \end{matrix}\right\} y^4 \left.\begin{matrix} + 14A^4 \\ - 21A^2B \\ + 3B^2 \\ + 6AC \\ - D \end{matrix}\right\} y^5 \left.\begin{matrix} - 42A^5 \\ + 84A^3B \\ - 28AB^2 \\ - 28A^2C \\ + 7BC \\ + 7AD \\ - E \end{matrix}\right\} y^6 + \ldots$$

Example.

Given the Logarithm $\bar{5}.039\ 9101\ 958$, required its natural number.

Tabular Number	Logarithms	Differences +	Differences −
$m' = 10962$	039 8897 977		
		396 164	
	039 9294 141		36
		396 128	
	039 9690 269		37
		396 091	
	040 0086 360		38
		396 055	
	040 0482 415		

Now $p = \bar{5}.039\ 9101\ 958$

$q = \underline{039\ 8897\ 977}$

$p - q = \underline{\underline{203\ 981}}$

$d' = +396\ 164$, $d'' = -36$

$\frac{1}{2}d'' = -18$, $d' - \frac{1}{2}d'' = +396\ 182$

$y = \frac{p-q}{d' - \frac{1}{2}d''} = \ldots\ldots 514\ 867$ } Reduced Arithmetically

$A = \frac{\frac{1}{2}d''}{d' - \frac{1}{2}d''} = \text{Log: } \bar{5}.658\ 2125$

A in this case is negative, the fraction from which it is derived having a negative numerator and a positive denomenator.

$y = \ldots\ldots\ldots\ldots\ldots\ldots\ 514\ 867$

$-Ay^2 = \ldots\ldots\ldots\ldots\ldots\ldots\ +12$

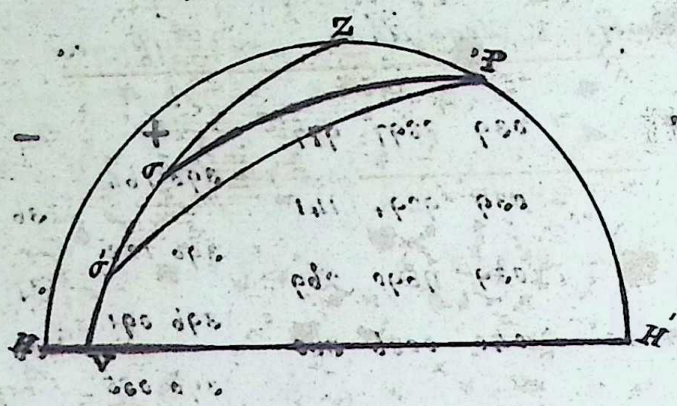
$+2A^2y^3 = \ldots\ldots\ldots\ldots\ldots\ldots$

$m' = \ldots\ldots\ldots\ldots\ldots\ldots\ 10962$

Required natural Number $\ldots\ldots\ldots\ldots$ $\underline{\underline{10962\ 514\ 879}}$

The same computation is applicable to the determination of an Arc from its sine, cosine, tangent, or cotangent.

Formula for correcting the transits of a high and low Star, both being situated to the South of the Equator.



HZPH́, is the Meridian.

Z, the Zenith.

P, the Pole.

V σ́ σ Z, is the vertical described by the Transit Instrument;

σ, σ́, being the positions of the two Stars thereon.

ZP = ½π − λ.

Pσ = ½π + d.

Pσ́ = ½π + d́.

Zσ = λ ± d

Zσ́ = λ + d́

ZPσ = δp

ZPσ́ = δṕ

Sin Pσ : Sin Zσ :: Sin σZP : Sin ZPσ

Sin Pσ́ : Sin Zσ́ :: Sin σ́ZP : Sin ZPσ́

Sin(½π+d) : Sin(λ+d) :: Sin(π−δA) : Sin δp

Sin(½π+d́) : Sin(λ+d́) :: Sin(π−δA) : Sin δṕ

Cos d : Sin(λ+d) :: δA : δp

Cos d́ : Sin(λ+d́) :: δA : δṕ

Hence.

$$\delta p = \frac{\sin(\lambda + d)}{\cos d} . \delta A$$

$$\delta p' = \frac{\sin(\lambda + d')}{\cos d'} . \delta A$$

Now subtracting δp, from $\delta p'$, the result is.

$$\delta p' - \delta p = \left\{ \frac{\sin(N+d')}{\cos d'} - \frac{\sin(N+d)}{\cos d} \right\} \delta A =$$

$$= \frac{\sin(N+d') \cdot \cos d - \sin(N+d) \cdot \cos d'}{\cos d' \cdot \cos d} \cdot \delta A$$

$$= \frac{\cos N \cdot \sin(d'-d)}{\cos d' \cdot \cos d} \cdot \delta A = t^2$$

Now put $\delta p' - \delta p = t^2$

Hence.

$$\delta A = \frac{t^2 \cdot \cos d' \cdot \cos d}{\cos N \cdot \sin(d'-d)}$$

Substituting this value for δA, in the formulæ for δp & $\delta p'$ there will arise.

$$\delta p = \frac{t^2 \cdot \cos d' \cdot \sin(N+d)}{\cos N \cdot \sin(d'-d)}$$

$$\delta p' = \frac{t^2 \cdot \cos d \cdot \sin(N+d')}{\cos N \cdot \sin(d'-d)}$$

To affix signs to δp & $\delta p'$

(1) When the Transit Instrument has eastern deviation, $(t+\delta p)$ & $(t'+\delta p')$ will obviously be the corrected times of the transits. Now suppose the high Star to transit after the low Star, then $(t+\delta p)-(t'+\delta p')$, will be the interval elapsed between the two Transits; that interval will evidently be equivalent to $(R-R')$. That is to say.

$$(t+\delta p) - (t'+\delta p') = (R-R')$$

Hence.

$(\delta p' - \delta p) = (t-t') - (R-R')$. Which is a positive quantity, whence it follows that when the culmination of the high star is subsequent to that of the low star, and $(t-t') > (R-R')$ the transit Instrument has eastern deviation.

(2) Again suppose the low star to come to the meridian after the star, then by following a process of reasoning similar to that given there will result.

$$(t' + \delta p') - (t + \delta p) = (R' - R)$$

or

$$(\delta p' - \delta p) = (R' - R) - (t' - t)$$

That is to say: the Transit Instrument will have eastern deviation when $(R' - R) > (t' - t)$ the culmination of the high star occurs first.

(3) Again when the Transit Instrument has western deviation $(t - \delta p)$ & $(t' - \delta p')$ will be the corrected times of the Transits. Now this is the position of the Instrument; there will be two cases analogous to those already treated of, for the other position of the Instrument. Take now the first case, in which the high star comes to the Meridian after the low star.

$$(t - \delta p) - (t' - \delta p') = (R - R')$$

or

$$(\delta p' - \delta p) = (R - R') - (t - t')$$

That is supposing the high star to culminate after the low star then if $(R - R') > (t - t')$ then the deviation of the Instrument is west.

(4) By a similar reasoning it will appear, that when the low star transits after the high star:

$$(t' - \delta p') - (t - \delta p) = (R' - R)$$

which will give

$(\delta p' - \delta p) = (t' - t) - (R' - R)$

That is to say the high star transitting first and also $(t' - t)$ being $> (R' - R)$ the Instrument will have western deviation.

(5) In the preceding investigation $\delta p'$ is taken $> \delta p$. This is obviously true since $\delta p'$ appertains to the low star.

On the Method of deriving the radius of curvature from the normal of the Axis Minor.

Now.

$$V = a\,(1-e^2 \sin^2 \lambda)^{-\frac{1}{2}}$$

$$\therefore \frac{V}{a} = (1-e^2 \sin^2 \lambda)^{-\frac{1}{2}}$$

or by cubing both sides

$$\frac{V^3}{a^3} = (1-e^2 \sin^2 \lambda)^{-\frac{3}{2}}$$

Now multiply both sides by $a(1-e^2)$ and there will result

$$a(1-e^2)\,(1-e^2 \sin^2 \lambda)^{-\frac{3}{2}} \text{ or } R = \frac{(1-e^2)V^3}{a^2}$$

Which is the expression for the radius of curvature in terms of the Normal.

The logarithmic value of the expression is as follows.

$$\mathrm{L}V = \text{Constant } \mathrm{L}\,\frac{(1-e^2)}{a^2} + 3\,\mathrm{L}\,V$$

Constant $\mathrm{L}\,\frac{(1-e^2)}{a^2}$ computed

$\mathrm{L}\,(1-e^2) =$	7·997 1076 098
$2\,\mathrm{L}\,a =$	14·641 2450 790
difference =	15·355 8625 308

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N°1 for Latitude

$$\Delta\lambda = \left\{ \begin{array}{l} -\cos A \cdot \frac{\varpi}{r} \\ -\frac{\tan\lambda}{2} \cdot \sin^2 A \cdot \frac{\varpi^2}{ry} \\ -\frac{3e^2}{2(1-e^2)} \cdot \sin\lambda \cdot \cos\lambda \cdot \cos^2 A \cdot \frac{\varpi^2}{ry} \\ +\frac{1+3\tan^2\lambda}{2.3} \cdot \sin^2 A \cdot \cos A \cdot \frac{\varpi^3}{ry^2} \end{array} \right\} \cdot \text{Cosec } i''$$

N°2 for Longitude

$$\Delta L = \left\{ \begin{array}{l} -\sec\lambda \cdot \sin A \cdot \frac{\varpi}{y} \\ +\tan\lambda \cdot \sec\lambda \cdot \sin A \cdot \cos A \cdot \frac{\varpi^2}{y^2} \\ -\frac{(1+3\tan^2\lambda)\cdot\sec\lambda}{3} \cdot \sin A \cdot \cos^2 A \cdot \frac{\varpi^3}{y^3} \\ +\frac{\tan^2\lambda \cdot \sec\lambda}{3} \cdot \sin^3 A \cdot \frac{\varpi^3}{y^3} \end{array} \right\} \text{Cosec}$$

N°3 for Azimuth

$$B = \Phi + A + \left\{ \begin{array}{l} -\tan\lambda \cdot \sin A \cdot \left(\frac{\varpi}{y}\right) \\ +\frac{1}{2}\left\{1+2\tan^2\lambda + \frac{e^2\cos^2\lambda}{(1-e^2)}\right\} \cdot \sin A \cdot \cos A \cdot \frac{\varpi^2}{y^2} \\ -\left(\frac{4}{3} + \tan^2\lambda\right) \cdot \tan\lambda \cdot \sin A \cdot \cos^2 A \cdot \frac{\varpi^3}{y^3} \cdot \\ +\frac{1+2\tan^2\lambda}{2} \cdot \frac{\tan\lambda}{3} \cdot \sin^3 A \cdot \frac{\varpi^3}{y^3} \end{array} \right\}$$

On the Method of deriving the constants used in the formula for the deduction of the Latitudes, Longitudes, and Azimuths of the Trigonometrical Stations

Constant for Part 1st of Latitude Computed

$$P' = \frac{Cosec\ 1''}{r}$$

$$P = L\ Cosec\ 1'' + A.C.L\ r$$

Constant for Part 1st of Longitude Computed

$$P' \times Q' \times Sec\ \lambda = \frac{Sec\ \lambda . Cosec\ 1''}{y}$$

Hence

$$Q' = \frac{Cosec\ 1''}{y} \times \frac{r}{Cosec\ 1''} = \frac{r}{y}$$

$$Q = L\ r + A.C.L\ y$$

Constant for Part 1st of Azimuth Computed

$$P' \times Q' \times Sec\ \lambda \times Sin\ \lambda = \frac{tan\ \lambda . Cosec\ 1''}{y}$$

Constant for Part 2nd of Latitude Computed

$$P' \times Q' \times Sec\ \lambda \times Sin\ \lambda \times R = \frac{tan\ \lambda}{2} \times \frac{Cosec\ 1''}{r.y}$$

Wherefore

$$R' = \frac{tan\ \lambda . Cosec\ 1''}{2 r y} \times \frac{y}{tan\ \lambda . Cosec\ 1''} = \frac{1}{2r}$$

$$R = A.C.L\ 2 + A.C.L\ r$$

Constant for Part 2nd of Longitude Computed

$$P' \times Q' \times \operatorname{Sec}\lambda \times \operatorname{Sin}\lambda \times R' \times S' = \frac{\tan\lambda \cdot \operatorname{Sec}\lambda \cdot \operatorname{Cosec} 1''}{\gamma^2}$$

$$S' = \frac{\tan\lambda \cdot \operatorname{Sec}\lambda \cdot \operatorname{Cosec} 1''}{\gamma^2} \times \frac{2\gamma\gamma}{\tan\lambda \cdot \operatorname{Cosec} 1''} = 2 \operatorname{Sec}\lambda \times \frac{\gamma}{\gamma_1}$$

$$S = \mathrm{L}\,2 + \mathrm{L} \operatorname{Sec}\lambda + Q.$$

Constant for Part 2nd of Azimuth Computed

$$P' \times Q' \times \operatorname{Sec}\lambda \times \operatorname{Sin}\lambda \times R' \times S' \times T' = t \cdot \left\{ 1 + \frac{e^2 \cdot \operatorname{Cos}^2\lambda}{2 \cdot (1-e^2) \cdot t} \right\} \cdot \frac{\operatorname{Cosec} 1''}{\gamma^2}$$

In which $t = \frac{1}{2}(1 + 2\tan^2\lambda)$

$$T' = t \left\{ 1 + \frac{e^2 \cdot \operatorname{Cos}^2\lambda}{2(1-e^2)t} \right\} \times \frac{\operatorname{Cosec} 1''}{\gamma^2} \times \frac{\gamma^2}{\tan\lambda \cdot \operatorname{Sec}\lambda \cdot \operatorname{Cosec} 1''} =$$

$$t \left\{ 1 + \frac{e^2 \cdot \operatorname{Cos}^2\lambda}{2(1-e^2)t} \right\} \cdot \operatorname{Cot}\lambda \cdot \operatorname{Cos}\lambda$$

$$T = \mathrm{L}\,t + \mathrm{L} \operatorname{Cot}\lambda + \mathrm{L} \operatorname{Cos}\lambda + \frac{me^2 \cdot \operatorname{Cos}^2\lambda}{2(1-e^2)t}$$

Constants for Part 3rd of Latitude Computed

$$P' \times Q' \times \operatorname{Sec}\lambda \times \operatorname{Sin}\lambda \times R' \times S' \times T' \times V' = \frac{3e^2}{4(1-e^2)} \operatorname{Sin} 2\lambda \cdot \frac{\operatorname{Cosec} 1''}{\gamma\gamma}$$

$$V' = \frac{3e^2}{4(1-e^2)} \cdot \operatorname{Sin} 2\lambda \cdot \frac{\operatorname{Cosec} 1''}{\gamma\gamma} \times \frac{\gamma}{\operatorname{Cosec} 1''} \times \frac{1}{t \cdot \left(1 + \frac{e^2 \cdot \operatorname{Cos}^2\lambda}{2(1-e^2)t}\right)} =$$

$$\frac{3e^2}{4(1-e^2)} \cdot \operatorname{Sin} 2\lambda \cdot \frac{\gamma}{\gamma\gamma} \times t' \cdot \left\{ 1 + \frac{e^2 \cdot \operatorname{Cos}^2\lambda}{2(1-e^2)t} \right\}$$

$$V = \text{Constant } \mathrm{L}\left\{ \frac{3e^2}{4(1-e^2)} \right\} + \mathrm{L} \operatorname{Sin} 2\lambda - Q - \mathrm{L}\,t - \frac{me^2 \cdot \operatorname{Cos}^2\lambda}{2(1-e^2)t}$$

Constant for Part 2nd of Azimuth Computed.

$$P' \times Q' \times \operatorname{Sec}\lambda \times \operatorname{Sin}\lambda \times R' \times S' \times T' = 2\alpha \times \frac{\operatorname{Cosec} i}{\gamma^2}$$

wherein $\alpha = \frac{1}{4}\left\{1 + 2\tan^2\lambda + \frac{e^2 . \operatorname{Cos}^2\lambda}{(1-e^2)}\right\}$

$$T' = \frac{\gamma^2}{\tan\lambda . \operatorname{Sec}\lambda . \operatorname{Cosec} i} \times \frac{2\alpha . \operatorname{Cosec} i}{\gamma^2} = 2\alpha . \operatorname{Cot}\lambda . \operatorname{Cos}\lambda .$$

$$T = L.2 + L.\alpha + L.\operatorname{Cot}\lambda + L.\operatorname{Cos}\lambda$$

Constant for Part 3rd of Latitude Computed

$$P' \times Q' \times \operatorname{Sec}\lambda \times \operatorname{Sin}\lambda \times R' \times S' \times T' \times V' = \frac{3e^2}{2(1-e^2)} . \operatorname{Sin}\lambda . \operatorname{Cos}\lambda \times \frac{\operatorname{Cosec} i}{r\gamma}$$

$$V' = \frac{3e^2}{2(1-e^2)} . \operatorname{Sin}\lambda . \operatorname{Cos}\lambda . \frac{\operatorname{Cosec} i}{r\gamma} \times \frac{\gamma^2}{2\alpha \operatorname{Cosec} i} = \frac{3e^2}{4(1-e^2)} . \operatorname{Sin}\lambda . \operatorname{Cos}\lambda \times \frac{\gamma}{r} \times \frac{1}{4\alpha}$$

$$V = \text{Constant } L\,\frac{3e^2}{4(1-e^2)} + L.\operatorname{Sin}\lambda + L.\operatorname{Cos}\lambda + \text{A.C. of } Q + \text{A.C. of } L\,\alpha$$

Constant for Part 5th of Longitude Computed.

$$P' \times Q' \times \operatorname{Sec}\lambda \times \operatorname{Sin}\lambda \times R' \times S' \times T' \times V' \times U' = \left(\tfrac{1}{3} + \tan^2\lambda\right) \operatorname{Sec}\lambda \times \frac{\operatorname{Cosec} i}{\gamma^3}$$

$$U' = \left(\tfrac{1}{3} + \tan^2\lambda\right) \operatorname{Sec}\lambda . \frac{\operatorname{Cosec} i}{\gamma^3} \times \frac{2(1-e^2)}{3e^2} . \operatorname{Cosec}\lambda . \operatorname{Sec}\lambda \times \frac{r\gamma}{\operatorname{Cosec} i} =$$

$$\left(\tfrac{1}{3} + \tan^2\lambda\right) \operatorname{Sec}^2\lambda . \operatorname{Cosec}\lambda \times \frac{2(1-e^2)}{3e^2} \times \frac{r}{\gamma^2}$$

$$U = \text{Constant } L\,\frac{2(1-e^2)}{3e^2} + Q + \text{A.C. of } L\,\gamma + L\left(\tfrac{1}{3} + \tan^2\lambda\right) + 2L\operatorname{Sec}\lambda + L\operatorname{Cosec}\lambda$$

Constant for Part 3rd of Longitude Computed

$$P' \times Q' \times \text{Sec } \lambda \times \text{Sin } \lambda \times R' \times S' \times T' \times V' \times U' = \frac{(1+3\tan^2\lambda)\,\text{Sec }\lambda}{3} \cdot \frac{\text{Cosec } i'}{y^3}$$

$$U' = \frac{(1+3\tan^2\lambda).\text{Sec }\lambda}{3} \cdot \frac{\text{Cosec } i'}{y^3} \times \frac{4(1-e^2)}{3e^2} \cdot \frac{1}{\sin 2\lambda} \cdot \frac{n\,y}{\text{Cosec } i} =$$

$$\frac{4(1-e^2)}{3e^2} \cdot \frac{1+3\tan^2\lambda}{3} \cdot \text{Sec}^2\lambda \cdot \text{Cosec}\,\lambda \times \frac{n}{y^2}$$

$$U = \text{Constant } L \frac{4(1-e^2)}{3e^2} + L\left(\frac{1+3\tan^2\lambda}{3}\right) + 2L.\text{Sec }\lambda + L\,\text{Cosec}\,\lambda + Q + A.C.L\,y$$

Constant for Part 3rd of Azimuth Computed

$$P' \times Q' \times \text{Sec } \lambda \times \text{Sin } \lambda \times R' \times S' \times T' \times V' \times U' \times W' = \left(\frac{5}{6} + \tan^2\lambda\right) \cdot \tan\lambda \cdot \frac{\text{Cosec } i'}{y^3}$$

$$W' = \left(\frac{5}{6} + \tan^2\lambda\right) \cdot \tan\lambda \cdot \frac{\text{Cosec } i''}{y^3} \times \frac{}{\frac{1}{3}(1+3\tan^2\lambda) \cdot \text{Sec }\lambda : \text{Cosec } i} =$$

$$\frac{\left(\frac{5}{6} + \tan^2\lambda\right) \cdot \text{Sin }\lambda}{\frac{1}{3}(1+3\tan^2\lambda)}$$

$$W = L\left(\frac{5}{6} + \tan^2\lambda\right) + L\,\text{Sin}\,\lambda + A.C.L\,\tfrac{1}{3}(1+3\tan^2\lambda)$$

Constant for Part 4th of Latitude Computed

$$P' \times Q' \times \text{Sec } \lambda \times \text{Sin}\,\lambda \times R' \times S' \times T' \times V' \times U' \times W' \times X' = \frac{1+3\tan^2\lambda}{6} \times \frac{\text{Cosec } i'}{n' y^2}$$

$$X' = \frac{(1+3\tan^2\lambda)}{6} \cdot \frac{\text{Cosec } i'}{} \times \frac{y^3}{\left(\frac{5}{6} + \tan^2\lambda\right) \cdot \tan\lambda \cdot \text{Cosec } i'} =$$

$$\frac{\frac{1}{6}(1+3\tan^2\lambda)}{6} \cdot \frac{V}{n'} \cdot \left(\frac{5}{6} + \tan^2\lambda\right)^{-1} \cdot \text{Cot}\,\lambda$$

$$X = L\,\tfrac{1}{3}(1+3\tan^2\lambda) + A.C.L\,2 + L\,\text{Cot}\,\lambda - L\left(\frac{5}{6} + \tan^2\lambda\right) - Q$$

Constant for Part 4th of Longitude Computed

$$P' \times Q' \times \sec\lambda \times \sin\lambda \times R' \times S' \times T' \times V' \times U' \times W' \times X' \times Y' = \frac{\tan^2\lambda . \sec\lambda}{3}$$

$$Y' = \frac{\tan^2\lambda . \sec\lambda}{3} \cdot \frac{\text{Cosec } i''}{Y^3} \times \frac{6}{1+3\tan^2\lambda} \cdot \frac{r^2 Y^3}{\text{Cosec } i''} =$$

$$\frac{\frac{2}{3}\tan^2\lambda . \sec\lambda}{\frac{1}{3}(1+3\tan^2\lambda)} \cdot \frac{r}{Y}$$

$$Y = L\left(\frac{2}{3}\right) + 2L\tan\lambda + L\sec\lambda - L\,\frac{1}{3}(1+3\tan^2\lambda) + Q$$

Constant for Part 4th of Azimuth Computed

$$P' \times Q' \times \sec\lambda \times \sin\lambda \times R' \times S' \times T' \times V' \times U' \times W' \times X' \times Y \times Z =$$

$$\frac{1+\tan^2\lambda}{2} \cdot \frac{\tan\lambda}{3} \cdot \frac{\text{Cosec } i''}{Y^3}$$

$$Z' = \frac{1}{2}(1+\tan^2\lambda) \cdot \frac{\tan\lambda}{3} \cdot \frac{\text{Cosec } i''}{Y^3} = \frac{3Y^3}{\tan^2\lambda . \sec\lambda . \text{Cosec } i''} =$$

$$\frac{\frac{1}{2}(1+\tan^2\lambda)}{\tan\lambda . \sec\lambda}$$

$$Z = L + \left(1+\tan^2\lambda\right) + L.\cot\lambda + L.$$

$$Z_4 = L\,t - L\tan\lambda - L\sec\lambda = T - \frac{m e^2 . \cos^2\lambda}{2.(1-e^2)}$$

Memorandum regarding the deduction of Colonel Everests Formulæ for the computation of terrestrial Latitude.

Let ϖ be a line on the surface of the earth, supposed to be spherical, and also let A be the azimuth of that line taken from one of its extremities A.

Calling S the Meridional distance corresponding to these elements; it will be found that

$$S = -\cos A\,\varpi - \frac{\tan\lambda.\sin^2 A}{2}\cdot\frac{\varpi^2}{\chi} + \frac{\sin A.\sin 2A\,(1+3\tan^2\lambda)}{3.4}\cdot\frac{\varpi^3}{\chi^2} + \ldots\ldots (1)$$

This is demonstrated at page CLXIII Colonel Everests Printed Work

Squaring and Cubing (1) there will result.

$$S^2 = \cos^2 A\,\varpi^2 + \tan\lambda.\sin^2\lambda.\cos A.\frac{\varpi^3}{\chi} + \ldots\ldots (2)$$

$$S^3 = -\cos^3 A.\varpi^3 + \ldots\ldots (3)$$

Now S is a Meridional distance and to convert it to an arc of Latitude in the assumption that the earth is a spheroid, the following series must be resorted to.

$$(\chi-\lambda) = \frac{1}{\nu}\cdot S - \frac{3\varepsilon^2.\sin 2\lambda}{2.2.(1-\varepsilon^2)\,\chi\nu}\cdot S^2 - \frac{3\varepsilon^2.\cos 2\lambda}{2.3(1-\varepsilon^2)\,\chi\nu}\cdot S^3 + \ldots\ldots (4)$$

Substitute in (4) the values of S, S^2, S^3, found before and there will arise.

$$(\chi-\lambda) = -\cos A\cdot\frac{\varpi}{\nu} - \frac{\tan\lambda.\sin^2 A}{2}\cdot\frac{\varpi^2}{\chi\nu}$$

$$+\frac{\sin A.\sin 2A.(1+3\tan^2 A)}{3.4}\cdot\frac{\varpi^3}{\nu\chi^2}$$

$$+\frac{3\varepsilon^2.\sin 2\lambda.\cos^2 A}{4(1-\varepsilon^2)}\cdot\frac{\varpi^3}{\chi\nu} + \ldots\ldots$$

Now the fourth term in the expanded expression for $(\lambda - \lambda)$ is $\frac{se^2 . \sin 2\lambda . \cos^2 A}{4.(1-e^2)} . \frac{\varpi}{\gamma r}$, This is the value accordingly given at p. 369 Vol 7 Part 2nd General Report. which see.

In Colonel Everests Printed work p. CLX the fourth term is put equivalent to $\frac{se^2 . \sin 2\lambda . \cos^2 A}{4} . \frac{\varpi^2}{\gamma r}$ which is likewise correct within the limits of accuracy possessed by the formul as the rejected term $- \frac{se^4 . \sin 2\lambda . \cos^2 \lambda}{4} . \frac{\varpi}{\gamma r}$ is of the forth order not taken into account by Colonel Everest.

Computation illustrating the precept for the deduction of the difference of Height between two Trigonometrical Stations ~~Cald~~ second deputy in Chief Computer's communication No. 49 dated 11th December 1849 to the address of the Surveyor General of India.

Required the difference of Height between Dhoiwala & Snowy Peak β

Suppose A to be Dhoiwala and B to be Snowy Peak β -

Geodetic distance from Dhoiwala to Snowy Peak β ...	Log....	5.5571202
Reduction to the Level of Dhoiwala ...		−636
Distance at the Height of Dhoiwala ...	Log...	5.5571838
Subtended ∠ 2°. 49′. 11″.7 ...	Log Sine...	$\bar{2}$.6919383
Vertical ∠ at Snowy Peak β. 3°. 18′. 48″ ...	Log. Sec....	0.0007266
Difference of Height = 17776.6 Feet ...	Log.	4.2498487

Again take A to be Snowy Peak β & B to be Dhoiwala

Geodetic distance from Snowy Peak β. to Dhoiwala ...	Log..	5.5571202
Reduction to the Level of Snowy Peak β ...		−4325
Distance at the Height of Snowy Peak β ...	Log.	5.5575527
Subtended ∠ 2°. 49′. 11″.7 ...	Log Sine.	$\bar{2}$.6919383
Vertical ∠ at Dhoiwala 2°. 19′. 35″.34 ...	Log Sec..	0.0003581
Difference of Height = 17776.6 Feet ...	Log...	4.2498491

No difference in the Results

On the method of interpolating for the rate of an astronomical Clock.

Suppose on a given day an astronomical Clock has the rate r.

Now let $(n-1)$ days pass over without any fresh determination of the rate. On the nth day suppose an observation is taken showing a great alteration in the error of the Clock.

Let e be the increment of error generated since the first observation was made, then $\frac{e}{n}$ is obviously the mean rate for the interval n.

Now $\frac{e}{n}$ may or may not agree with r. If it does not agree and still if it is made use of for the intermediate period it will follow that during the first twenty-four hours after r was determined, that term changed into $\frac{e}{n}$, and that afterward for the remaining $(n-1)$ days, the new rate $\frac{e}{n}$ remained constant. It is evident that this view of the matter is inadmissible because no good Astronomical Clock would act in the way here described. If its rate varies, it does so slowly and gradually. It cannot suddenly change from one value to another during one day and then remain constant afterward.

Proceeding therefore upon the probable assumption of the uniform alteration of the rate of the Clock, I will suppose that $r+\delta r$, $r+2\delta r$, $r+3\delta r$, ... $r+n\delta r$, are the rates for the several days intervening between the 1st and 2nd observations δr being the daily variation of the rate.

To determine δr, add together all the rates, the sum will obviously be equivalent to e. That is to say.

$$nr + \tfrac{1}{2} n.(n+1).\delta r = e$$

Wherefore.

$$\delta r = \frac{e - nr}{\frac{1}{2} n.(n+1)}$$

δr, being known, the rates for the several intermediate days can be easily computed.

Example

On the 7th December 1849, the error and rate of Thwaites were as follows.

Error..... 0h. 12m. 44s·36. Fast

Rate..... 8s·10. Gaining

On the 13th December again

Error..... 0h. 13m. 24s·49. Fast

In this case.

$e = 40^s \cdot 13$

$r = 8^s \cdot 10$

$n = 6$ days

Wherefore.

$$\delta r = \frac{40^s \cdot 13 - 48^s \cdot 60}{2} = -1^s \cdot 403$$

The errors and rates of Thwaites. therefore from the 8th to the 13th are as follows.

December. 8th	Rate =	$7^s \cdot 70$	Error 0h. 12m. 52s·06
9th	"	$7^s \cdot 29$	" 0h. 12m. 59s·35
10th	"	$6^s \cdot 89$	" 0h. 13m. 6s·24
11th	"	$6^s \cdot 49$	" 0h. 13m. 12s·73
12th	"	$6^s \cdot 09$	" 0h. 13m. 18s·82
13th	"	$5^s \cdot 68$	" 0h. 13m. 24s·50

In a triangle, given an angle and the two sides adjacent thereto, one of which being very small in comparison to the other, it is required to compute the third side.

In this deduction, the characters used are as follow

a = Longer Side

b = Shorter Side

C = Included angle

c = Third Side sought

The fundamental formula in Plane-Trigonometry expressing the relation between the sides and an angle of a Triangle is this:

$$c^2 = a^2 + b^2 - 2ab.\cos C$$

Hence..... $c = a\left\{1 + \frac{b^2}{a^2} - 2.\frac{b}{a}.\cos C\right\}^{\frac{1}{2}}$. Which expanded by the Binomial Theorem and reduced becomes.

$$c = a - b.\cos C + \frac{b^2}{2a}.\sin^2 C + \text{.....}$$

In this expression, the first term being given, the second term is that which will require computation, the third & others being considered as rejectaneous, on account of their minuteness. Here it ought to be remembered that, $b.\cos C$ is negative, as it is in the formula when $\angle C$ is in the 1st Quadrant and that it becomes positive when $\angle C$ is in the 2nd Quadrant

1st Example

Darot to Sidpur.... $a = 49088$

Darot to Jorati.... $b = 11.42$ Log. 1.0576661

$\angle$ Darot..... $C = 39^\circ.17'.33''$ Cos. $\bar{1}.8886992$

-8.8 ... Log. 0.9463563

$a = 49088$

Required value of $c = 49079.2$

2nd Example

Darot to Dhar. $a = 57084$

Darot to Jorat. $b =$ 11·42. Log. 1·0576661

$\angle$ Darot. $C = 75.48.6.$ Cos. T·3896607

$= 2.8.$ Log. 0·4473268

$a = 57084$

Required value of $c = 57081·2$

$=$

The expression for the Terrestrial excentricity is this: $e^2 = \frac{a^2-b^2}{a^2}$. which gives $1-e^2 = \frac{b^2}{a^2}$. If now b & a were to vary, $(1-e^2)$ must likewise vary. Now let the variations of b & a & $(1-e^2)$ be respectively represented by δb, δa, & $\delta(1-e^2)$. we shall have as follows

$$(1-e^2)+\delta(1-e^2) = \frac{(b+\delta b)^2}{(a+\delta a)^2} = \frac{b^2}{a^2}\cdot\frac{(1+\frac{\delta b}{b})^2}{(1+\frac{\delta a}{a})^2} =$$

$$\frac{b^2}{a^2}\left\{1+\frac{2\delta b}{b}+\frac{\delta b^2}{b^2}\right\}\cdot\left\{1-\frac{2\delta a}{a}+\frac{3\delta a^2}{a^2}+\text{&c}\right\} =$$

$$\frac{b^2}{a^2}\left\{1+\frac{2\delta b}{b}+\frac{\delta b^2}{b^2}-\frac{2\delta a}{a}-\frac{4\delta a.\delta b}{a.b}+\frac{3\delta a^2}{a^2}+\text{&c}\right\} =$$

$$\frac{b^2}{a^2}+\frac{2b.\delta b}{a^2}-\frac{2b^2.\delta a}{a^3}+\frac{\delta b^2}{a^2}-\frac{4b.\delta a.\delta b}{a^3}+\frac{3b^2.\delta a^2}{a^4}$$

The object of this investigation is this: Suppose $(1-e^2)$ is computed for one set of values of a & b. it will be easy to deduce $(1-e^2)$ for any other set. For calling δa the difference between the two values of a. & δb the difference between the two values of b. then we get.

$$\delta(1-e^2) = \frac{2b.\delta b}{a^2}-\frac{2b^2\delta a}{a^3}+\frac{\delta b^2}{a^2}-\frac{4b.\delta a.\delta b}{a^3}+\frac{3b^2.\delta a^2}{a^4}$$

Which terms are of easy computation by Logarithms. $\delta(1-e^2)$ applied to $(1-e^2)$ will furnish the new value for the latter term corresponding to the altered values of a & b.

To compute the position of the intersection of the parallel from Haliana with the Halianpur Meridian.

The perpendicular from Haliana meets the Halianpur Meridian at IV p. Now Haliana and IV p. not being in the same parallel they cannot have the same latitude. It is required to compute the difference of latitude between them. For this purpose, suppose the Meridians of Halianpur and Haliana extended until they meet at the Pole. Here is a right angled triangle formed, of which the three sides are the perpendicular distance of IV p. to Haliana and the Colatitudes of these two points.

λ = Latitude of Haliana.

$\lambda + \Delta\lambda$ = Latitude of IV p.

p = Distance from Haliana to IV p.

Hence by Napier's rule of circular parts.

$\sin \lambda = \cos p \cdot \sin(\lambda + \Delta\lambda)$

Now since p & $\Delta\lambda$ are very small we may safely substitute as follows.

$\cos p = 1 - \frac{1}{2} p^2$

$\sin(\lambda + \Delta\lambda) = \sin\lambda + \cos\lambda \cdot \Delta\lambda$.

Making these substitutes there will result.

$\Delta\lambda = \frac{1}{2} p^2 \cdot \tan\lambda$.

Now these terms are not homologous and that they may be made so the value of the Radius must be restored where it is necessary. This done the formula becomes.

$\Delta\lambda = \frac{p^2 \cdot \tan\lambda}{2r}$.

Example

p = 997·792 Feet ..... { Log. 2·9990401 / Log. 2·9990401

λ = 29° 30′ 55″ ..... Log Tan 9·7629122

Normal to the Axis Minor ..... A. C. of log ν = 7·6799274

A. C. of log. 2 ..... 1·6989700

$\Delta\lambda$ in feet 0·8117 ..... 2·0618096

The arc Kalianpur to $\overset{N}{p}$. being augmented by ΔN, a point will be obtained, which will have the same Latitude as N. Kaliana.

Memo:

On the method of determining the deflection of the Plummet at the station of observation.

Let Z, be the true zenith and $\acute{Z}$ the = ∠ of the zenith of the Instrument. Also let P, be the pole and R, the referring Lamp. Join Z $\acute{Z}$, $\acute{Z}$P, $\acute{Z}$P, ZP, ZR & $\acute{Z}$R by arcs of great circles. It is clear that.

1stly..... Z $\acute{Z}$, represents the whole deflection of the plummet ∠P$\acute{Z}$Z being the azimuth of the same.

2ndly..... $\acute{Z}$P, is the observed colatitude of the station of observation, ZP being the true colatitude thereof.

3rdly..... $\acute{Z}$R, is the observed zenith distance of R, ZR being the true zenith distance of the same.

Lastly..... ∠P$\acute{Z}$R, is the observed azimuth of R, ∠PZR, being the true azimuth thereof.

In this statement those are called observed elements which are referred to the plane of the Instrument, the true elements being those which are taken with reference to the plane of the horizon. Now there are three quantities of the latter class of which two, viz. side ZP, & ∠PZR, are derived from previous computation, and the third, viz. side ZR, may be determined in this way. Find by

Barometrical measurement the difference of height between R, and the station of observation. Convert that difference to arc, and take the complement thereof, which will be the value of ZR.

Now in the spherical triangle PZR, two sides ZP, & ZR, and the included angle ∠Z being given ∠ZPR, may be computed. Again in the spherical triangle PZR, sides ZP & ZR and the included angle ∠PZR being known ∠ZPR, may be deduced. And lastly take the spherical triangle ZPz, in which the ∠P, (which is equivalent to the difference between the two angles computed as directed above) being known, as likewise the sides ZP, & ZP, line Zz and the ∠PZz may be deduced.

In a spherical triangle, given two sides which are nearly equal, and the included angle which is very small, it is required to compute the third side.

$$\sin a . \sin b . \cos C + \cos a . \cos b - \cos c = 0 \quad \ldots\ldots (1)$$

Let $b = a + \delta a$.

Hence we get,

$$\sin a . \sin b = \sin^2 a . \cos \delta a + \tfrac{1}{2} \sin 2a . \sin \delta a .$$

$$\cos a . \cos b = \cos^2 a . \cos \delta a - \tfrac{1}{2} \sin 2a . \sin \delta a .$$

Making these substitutions in equation (1) there will results.

$$\left.\begin{array}{l} \sin^2 a . \cos \delta a . \cos C + \tfrac{1}{2} \sin 2a . \sin \delta a . \cos C \\ + \cos^2 a . \cos \delta a - \tfrac{1}{2} \sin 2a . \sin \delta a \\ - \cos . c \end{array}\right\} = 0 \quad \ldots\ldots (2)$$

Now.

$\sin^2 a . \cos \delta a . \cos C + \cos^2 a . \cos \delta a =$ (because $\cos^2 a = 1 - \sin^2 a$)

$- 2 \sin^2 a . \cos \delta a . \sin^2 \frac{1}{2} C + \cos \delta a = -2 \sin^2 a . \cos \delta a . \sin^2 \frac{1}{2} C + 1 - \sin^2 \frac{1}{2} \delta a$

Again.

$2 \sin 2a . \sin \delta a . \cos C - 2 \sin 2a . \sin \delta a = 4 \sin 2a . \sin \delta a . \sin^2 \frac{1}{2} C$

Also.

$- \cos c = -1 + 2 \sin^2 \frac{1}{2} c .$

Substitute these quantities in equation (2) we will have.

$$\sin^2 \tfrac{1}{2} c = \sin^2 \tfrac{1}{2} a + \sin^2 \tfrac{1}{2} C . \{ \sin^2 a . \cos \delta a + 2 \sin 2a . \sin \delta a \} \ldots\ldots (3)$$

Suppose the last decimal of a Logarithm to vary, it is required to compute the corresponding alteration in the natural number thereof.

Let $u = \mathrm{L}x$, or $x = a^u$

Now suppose u & x to vary the former becoming $u + \Delta u$ and the latter $x + \Delta x$. then.

$$x + \Delta x = a^{u + \Delta u} = x a^{\Delta u} = x (1 + A \Delta u + \tfrac{1}{2} A^2 \Delta u^2 + \tfrac{1}{6} A^3 \Delta u^3 + \ldots)$$

Hence.

$$\Delta x = x . A \Delta u + \tfrac{1}{2} x A^2 \Delta u^2 + \tfrac{1}{6} x A^3 \Delta u^3 + \ldots\ldots$$

In which A stands for the Hyperbolical Logarithm of a which is the base of the system.

suppose $a = 10$, then $A = 2 \cdot 3026851$,

Example 1.

Given Logarithm. $w = \bar{1}.3180633350$

Corresponding natural number $x = 20800$

variation in the given Logarithm: $\Delta w = 000\,000\,0005$

x	Log..	4·3180633
A	Log	0·3802157
Δw	Log..	$\bar{10}$·6989700
		$\bar{5}$·3792490
Variation in the natural numbers	$\Delta x =$	000023947

Suppose the radius of curvature to vary, it is required to compute the alteration consequent thereon in the value of the Terrestrial arc in seconds.

m = Value of any terrestrial arc in feet.

$\dot{m}$ = ——— Do: ——— Do: ——— in seconds.

r = Radius of curvature in feet.

Hence. $\dot{m} = \frac{m}{r} \cdot \text{Cosec } i$

Now let $\dot{m}$ & r to vary the former becoming $\dot{m} + \delta\dot{m}$ and the latter $r + \delta r$. Hence.

$$\dot{m} + \delta\dot{m} = \frac{m.\text{Cosec } i}{r + \delta r} = \frac{m.\text{Cosec } i}{r\left(1 + \frac{\delta r}{r}\right)} = \frac{\dot{m}}{\left(1 + \frac{\delta r}{r}\right)} =$$

$$\dot{m} - \frac{\dot{m}.\delta r}{r} + \frac{\dot{m}.\delta r^2}{r^2} \text{ ——————}$$

$$\delta\dot{m} = -\frac{\dot{m}.\delta r}{r} + \frac{\dot{m}.\delta r^2}{r^2} - \text{———}$$

Example

Radius of curvature $r = 20820068$ Feet ..... } A. C. of Log. 7.6815179.

Value of the Terrestrial arc in seconds $m'' = 19429.3562$ ... } —— Log. 4.2884504

variation in r, $\delta r = 1$ foot —— Log. 0.0000000

variation in the arc, $\delta m'' = .0009$ —— Log. 4.9699683

Col: Everest's old Constants computed.-

Feet

a = 209 229 31·80 ..... Log 7·3206225396

b = 208 533 74·58 ..... Log 7·3191763443

$\frac{b}{a}$ = 0·996 675 550 986 ..... Log 7·998563 8049

$(1+\frac{b}{a})$ = 1·996 675 550 986 Log 0·3003075 001

$(1-\frac{b}{a})$ = 0·003 324 449 014 Log $\bar{3}$·521 7196 767 (C)

Now $e^2 = (1+\frac{b}{a})\cdot(1-\frac{b}{a})$

Hence e^2 = .006 647 846 07.. Log $\bar{3}$·822 0271 770

$(1-e^2)$ = .993 352 153 93.. Log 7·997 1076 098

Also $1-e^2 = (\frac{b}{a})^2$

Hence $1-e^2$ = .993 352 153 93.. This agrees with the former value

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Col: Everest's new Constants computed.-

	Feet	
a =	209 209 02·48	Log 7·320 580 4181
b =	208 536 42·02	Log 7·319 181 9741
$\frac{b}{a}$ =	0·996 785 011733	Log $\bar{1}$· 998 601 4991
$(1+\frac{b}{a})$ =	1·996 785 011733	Log 0·300 331 3831
$(1-\frac{b}{a})$ =	0·003 214 988267	Log $\bar{3}$·507 179 3924

$e^2=(1+\frac{b}{a})\cdot(1-\frac{b}{a})$

e^2 =	·006 419 64038	Log $\bar{3}$·807 510 7002
$(1-e^2)$ =	·993 580 35962	Log $\bar{1}$·997 202 9981

Also $1-e^2=(\frac{b}{a})^2$

Hence $1-e^2$ = ·993 580 359 615 556 947 663 284 ...

which agrees with the former value.-

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Airy's Constants computed.

Feet

$a = 209\,237\,13$ ..... Log. 7.3206387544

$b = 208\,538\,10$ ..... Log. 7.3191854123

$\frac{b}{a} = 0.996659149358$ ..... Log. $\bar{1}$.9985466579

$(1+\frac{b}{a}) = 1.996659149358$ Log 0.3003039326

$(1-\frac{b}{a}) = 0.003340850642$ Log $\bar{3}$.5238570603

$$e^2 = (1+\frac{b}{a}) \cdot (1-\frac{b}{a})$$

$e^2 = .00667054000$ ..... Log. $\bar{3}$.8241609927

$1-e^2 = .99332946000$ ..... Log. $\bar{1}$.9970933159

Also $1-e^2 = (\frac{b}{a})^2$

Hence $1-e^2 = .99332945999901215181216 4$.

which accords with the former deduction

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Bessel's Constants computed.-

Feet

$a = 20922811.64$ Log. 7.3206200453

$b = 20853232.37$ Log. 7.3191733826

$\frac{b}{a} = 0.996674478020$ Log. $\bar{1}$.9985533374

$(1+\frac{b}{a}) = 1.996674478020$ Log. 0.3003072667

$(1-\frac{b}{a}) = 0.003325521980$ Log. $\bar{3}$.5218598227

$$e^2 = (1+\frac{b}{a}).(1-\frac{b}{a})$$

$e^2 = .00663998486$ Log. $\bar{3}$.8221670892

$1-e^2 = .99336001514$ Log. $\bar{1}$.9971066747

Also $1-e^2 = (\frac{b}{a})^2$

Hence $1-e^2 = .993360015136439463120400$

which accords with the former deduction

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen.

Terrestrial Arc between Kalianpur and $\bar{p}$. Converted into seconds with Colonel Everest's Old Constants.

Process No. 1

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Origin.

Latitude of Kalianpur = 24°. 7'. 11".262

$a = 20922931.8$ Feet ..... Log. $a = 7.3206225395$

$e^2 = .00663784607$ Log. $e^2 = \bar{3}.8220271770$

$1-e^2 = .99336215393$ Log. $(1-e^2) = \bar{1}.9971076098$

$r = 20818654.0860$ Feet ..... Log. $r = 7.3184526492$

$\nu =$ ———— Log. $\nu = 7.3208633728$

$\frac{\Delta s}{r} = \quad + \quad 19430''.6760$

$- \quad \frac{3e^2 . \sin 2\lambda \, \Delta s^2}{4(1-e^2)\,\nu\, r} = - \quad 6.8049$

$- \quad \frac{e^2 . \cos 2\lambda \, \Delta s^3}{2(1-e^2)\,\nu\, r^2} = - \quad 0.3816$

$+ \quad \frac{e^4 . \sin^2 2\lambda \, \Delta s^3}{2(1-e^2)^2\,\nu^2\, r} = + \quad 0.0021$

$+ \quad \frac{e^2 . \sin 2\lambda \, \Delta s^4}{4(1-e^2)\,\nu\, r^3} = + \quad 0.0201$

$+ \quad \frac{15e^4 . \sin 4\lambda \, \Delta s^4}{32(1-e^2)^2\,\nu^2\, r^2} = + \quad 0.0003$

$+ \quad \frac{e^2 . \cos 2\lambda \, \Delta s^5}{10(1-e^2)\,\nu\, r^4} = + \quad 0.0007$

Sum = 19423.5127 = 5°. 23'. 43".5127

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Halianpur and p^{IV}. Converted into seconds with Colonel Everest's Old Constants:

Process No. 2

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Middle Point.

Latitude of the Middle Point = 26°. 49'. 3".005

$r = 20826240 \cdot 0873$ Feet ..... Log. r = 7·3186108707

V = ———— Log. V = 7·3209161133

$\frac{\Delta s}{r}$ = + 19423".5984

$- \frac{e^2 . \cos 2\lambda \, \Delta s^3}{8(1-e^2) V r^2}$ = − 0·0849

$+ \frac{e^4 . \sin^2 2\lambda \, \Delta s^3}{8(1-e^2)^2 V^2 r}$ = + 0·0006

Sum = 19423".5141 = 5°. 23'. 43".5141

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Kalianpur and $p^{\#}$. Converted into π seconds with Colonel Everest's Old Constants.

Process No. 3

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Terminus.

Latitude of Kaliana = 29°. 30'. 54".747

$r = 20834376.5163$ Feet — Log. $r = 7.3137806084$

$v =$ — Log. $v = 7.3209726592$

$\frac{\Delta s}{r} =$ + 19416".0129

$+ \frac{3e^2 . \sin 2\lambda \, \Delta s^2}{4(1-e^2)\, v\, r} = +$ 7.8742

$- \frac{e^2 . \cos 2\lambda \, \Delta s^3}{2(1-e^2)\, v\, r^2} = -$ 0.2943

$+ \frac{e^4 . \sin^2 2\lambda \, \Delta s^3}{2(1-e^2)^2\, v^2\, r} = +$ 0.0028

$- \frac{e^2 . \sin 2\lambda \, \Delta s^4}{4(1-e^2)\, v\, r^3} = -$ 0.0231

$- \frac{15 e^4 . \sin 4\lambda \, \Delta s^4}{32(1-e^2)^2\, v^2\, r^2} = -$ 0.0003

$+ \frac{e^2 . \cos 2\lambda \, \Delta s^5}{10(1-e^2)\, v\, r^4} = +$ 0.0005

Sum = 19423.5127 = 5°. 23'. 43".5127

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Kalianpur and p.^{II} Converted into seconds with Colonel Everest's New Constants.

Process No. 1

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Origin.

Latitude of Kalianpur = 24°. 7'. 11".262

$a = 20920902.48$ Feet	Log. a =	7.3205804151
$e^2 = .00641964038$	Log. e^2 =	$\bar{3}.8075107002$
$1-e^2 = .99358035962$	Log. $(1-e^2)$ =	$\bar{1}.9972029981$
$r = 20820068.2950$ Feet	Log. r =	7.3184821498
γ = ————	Log. γ =	7.3208133273

$\frac{\Delta s}{r}$ =	+	19429".3562	+	= [illegible]	+
$-\frac{3e^2 . \sin 2\lambda \, \Delta s^2}{4(1-e^2)\gamma r}$ =	−	6.5801	−	= [illegible]	−
$-\frac{e^2 . \cos 2\lambda \, \Delta s^3}{2(1-e^2)\gamma r^2}$ =	−	0.3689	+	= [illegible]	+
$+\frac{e^4 . \sin^2 2\lambda \, \Delta s^3}{2(1-e^2)^2 \gamma^2 r}$ =	+	0.0020	−	= [illegible]	−
$+\frac{e^2 . \sin 2\lambda \, \Delta s^4}{4(1-e^2)\gamma r^3}$ =	+	0.0195	−	= [illegible]	−
$+\frac{15e^4 . \sin 4\lambda \, \Delta s^4}{32(1-e^2)^2 \gamma^2 r^2}$ =	+	0.0003	+	= [illegible]	+
$+\frac{e^2 . \cos 2\lambda \, \Delta s^5}{10(1-e^2)\gamma r^4}$ =	+	0.0007		= [illegible]	
Sum =		19422.4297	=	5°. 23'. 42".4297	

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Halianpur and p. Converted into seconds with Colonel Everest's New Constants.

Process No. 2

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Middle Point.

Latitude of the Middle Point = 26°. 49'. 3"·005

r = 20827405·0825 Feet..... Log. r = 7·3186351639

v = ———— Log. v = 7·3208643320

$\frac{\Delta S}{r}$ = + 19422"·5119

$- \frac{e^2 . \cos 2\lambda \, \Delta S^3}{8(1-e^2)\, v\, r^2}$ = − 0·0820

$+ \frac{e^4 . \sin^2 2\lambda \, \Delta S^3}{8(1-e^2)^2\, v^2\, r}$ = + 0·0006

Sum = 19422·4305 = 5°. 23'. 42"·4305

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Kalianpur and p^{II}: Converted into seconds with Colonel Everest's New Constants.

Process No. 3.

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Terminus.

Latitude of Kaliana = 29°. 30'. 54".747

$r = 20835274.0443$ Feet ..... Log. $r = 7.3187992171$ =

$\nu =$ ..... = ... Log. $\nu = 7.3209190164$ =

$\frac{\Delta s}{r} =$		+	19415.1765	+	=	
+	$\frac{3e^2 . \sin 2\lambda \Delta s^2}{4(1-e^2)\nu r} =$	+	7.5563	−	=	−
−	$\frac{e^2 . \cos 2\lambda \Delta s^3}{2(1-e^2)\nu r^2} =$	−	0.2846	+	=	+
+	$\frac{e^4 . \sin^2 2\lambda . \Delta s^3}{2(1-e^2)^2 \nu^2 r} =$	+	0.0026		=	
−	$\frac{e^2 . \sin 2\lambda . \Delta s^4}{4(1-e^2)\nu r^3} =$	−	0.0223			
−	$\frac{15e^4 . \sin 4\lambda \Delta s^4}{32(1-e^2)^2 \nu^2 r^2} =$	−	0.0003			
+	$\frac{e^2 . \cos 2\lambda \Delta s^5}{10(1-e^2)\nu r^4} =$	+	0.0005			
Sum =			19422.4287 =	5°. 23'. 42".4287		

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Kalianpur and $\overset{\text{IV}}{p}$. Converted into seconds with Airy's Constants.

Process No. 1.

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Origin.

Latitude of Kalianpur = 24°. 7'. 11".262

$a =$ 20923713 Feet	Log. $a =$	7·3206387544
$e^2 =$ ·00667054000	Log. $e^2 =$	$\bar{3}$·8241609927
$1-e^2 =$ ·99332946000	Log. $(1-e^2) =$	$\bar{1}$·9970933159
$r =$ 20818916·8478 Feet	Log. $r =$	7·3184581306
$\nu =$ ———	Log. $\nu =$	7·3208807745

$\frac{\Delta S}{r} =$	+	19430".4308	
$-\ \frac{3e^2 . \sin 2\lambda\ \Delta S^2}{4(1-e^2)\ \nu\ r} =$	−	6·8383	
$-\ \frac{e^2 . \cos 2\lambda\ \Delta S^3}{2(1-e^2)\ \nu\ r^2} =$	−	0·3834	
$+\ \frac{e^4 . \sin^2 2\lambda\ \Delta S^3}{2(1-e^2)^2\ \nu^2\ r} =$	+	0·0021	
$+\ \frac{e^2 . \sin 2\lambda\ \Delta S^4}{4(1-e^2)\ \nu\ r^3} =$	+	0·0202	
$+\ \frac{15e^4 . \sin 4\lambda\ \Delta S^4}{32(1-e^2)^2\ \nu^2\ r^2} =$	+	0·0003	
$+\ \frac{e^2 . \cos 2\lambda\ \Delta S^5}{10(1-e^2)\ \nu\ r^4} =$	+	0·0007	
Sum =		19423·2324 =	5°. 23'. 43".2324

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Kalianpur and $\overset{\text{II}}{p}$. Converted into seconds with Airys Constants.

Process No. 2.

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Middle Point.

Latitude of the Middle Point = 26°.49'.3".005

r = 20826540·3655 Feet ..... Log. r = 7·3186171324

ν = ———— Log. ν = 7·3209337751

$\frac{\Delta S}{r}$ = + 19423·3183

$-\frac{e^2 . \cos 2\lambda \Delta S^3}{8(1-e^2)\nu r^2}$ = − 0·0853

$+\frac{e^4 . \sin^2 2\lambda \Delta S^3}{8(1-e^2)^2 \nu^2 r}$ = + 0·0006

Sum = 19423·2336 = 5°.23'.43".2336

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Kalianpur and $\overset{\text{II}}{p}$. Converted into seconds with Airy's Constants.

Process No. 3

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Terminus.

Latitude of Kaliana = 29 . 30 . 54·747

$r = 20834717{\cdot}0784$ Feet..... Log. $r = 7{\cdot}3187876074$

$v =$ ———— Log. $v = 7{\cdot}3209906001$

$\frac{\Delta s}{r} = \quad + \quad 19415{\cdot}6955$

$+ \quad \frac{3e^2 . \sin 2\lambda \, \Delta s^2}{4(1-e^2)\, v\, r} = + \quad 7{\cdot}8525$

$- \quad \frac{e^2 . \cos 2\lambda \, \Delta s^3}{2(1-e^2)\, v\, r^2} = - \quad 0{\cdot}2957$

$+ \quad \frac{e^4 . \sin^2 2\lambda \, \Delta s^3}{2(1-e^2)^2\, v^2\, r} = + \quad 0{\cdot}0028$

$- \quad \frac{e^2 . \sin 2\lambda \, \Delta s^4}{4(1-e^2)\, v\, r^3} = - \quad 0{\cdot}0232$

$- \quad \frac{15e^4 . \sin 4\lambda \, \Delta s^4}{32(1-e^2)^2\, v^2\, r^2} = - \quad 0{\cdot}0003$

$+ \quad \frac{e^2 . \cos 2\lambda \, \Delta s^5}{10(1-e^2)\, v\, r^4} = + \quad 0{\cdot}0005$

Sum = $19423{\cdot}2321 = 5^{\circ} . 23' . 43''{\cdot}2321$

Computed & Compared by
Sreenath Seedar
& Radhanauth Sen

III

Terrestrial Arc between Kalianpur and p. Converted into seconds with Bessel's Constants.

Process No. 1

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Origin.

Latitude of Kalianpur = 24° . 7' . 11".262

a = 20922811·64 Feet		Log. a = 7·3206200453
e^2 = ·00663998486		Log. e^2 = $\bar{3}$·8221670892
$1-e^2$ = ·99336001514		Log. $(1-e^2)$ = $\bar{1}$·9971066747
r = 20818500·8569 Feet		Log. r = 7·3184494527
ν =		Log. ν = 7·3208609562
$\frac{\Delta B}{r}$ =	+ 1943″·8190	
$-\frac{e^2 \cdot \sin 2\lambda \, \Delta B^2}{4(1-e^2)\,\nu\, r}$ =	− 6·8072	
$-\frac{e^2 \cdot \cos 2\lambda \, \Delta B^2}{2(1-e^2)\,\nu\, r^2}$ =	− 0·3817	
$+\frac{e^4 \cdot \sin^2 2\lambda \, \Delta B^2}{2(1-e^2)^2\,\nu^2\, r}$ =	+ 0·0021	
$+\frac{e^2 \cdot \sin 2\lambda \, \Delta B^4}{4(1-e^2)\,\nu\, r^3}$ =	+ 0·0201	
$+\frac{15\,e^4 \cdot \sin 4\lambda \, \Delta B^4}{32(1-e^2)^2\,\nu^2\, r^2}$ =	+ 0·0003	
$+\frac{e^2 \cdot \cos 2\lambda \, \Delta B^5}{10(1-e^2)\,\nu\, r^4}$ =	+ 0·0007	
Sum =	1943″·6533 =	0° . 23' . 43″·6533

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Kalianpur and $\bar{p}$. Converted into seconds with Bessel's Constants.

Process No. 2

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Middle Point.

Latitude of the Middle Point = 26°. 49'. 3".005

r = 20826089·2616 Feet..... Log. r = 7·3186077255

V = ———————— Log. V = 7·3209137138

$\frac{\Delta S}{r}$ = + 19423"·7390

$-\frac{e^2 \cdot \cos 2\lambda \, \Delta S^3}{8(1-e^2)\, V\, r^2}$ = − 0·0849

$+\frac{e^4 \cdot \sin^2 2\lambda \, \Delta S^3}{8(1-e^2)^2\, V^2\, r}$ = + 0·0006

Sum = 19423·6547 = 5°. 23'. 43".6547

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Terrestrial Arc between Kalianpur and p. Converted into seconds with Bessel's Constants.

Process No. 3

Consists in making this reduction with the Radius of Curvature and the Normal to the Axis Minor for the Latitude of the Terminus.

Latitude of Kaliana = 29°. 30'. 54".747

r = 20834228·2663 Feet ..... Log. r. = 7·3187774181

ν = ———— Log. ν. = 7·3209702780

$\frac{\Delta s}{r}$ =	+	19416"·1510	
+ $\frac{3e^2 . \sin 2\lambda \, \Delta s^2}{4(1-e^2)\,\nu\, r}$ =	+	7·8169	
− $\frac{e^2 . \cos 2\lambda \, \Delta s^3}{2(1-e^2)\,\nu\, r^2}$ =	−	0·2944	
+ $\frac{e^4 . \sin^2 2\lambda \, \Delta s^3}{2(1-e^2)^2\,\nu^2\, r^2}$ =	+	0·0028	
− $\frac{e^2 . \sin 2\lambda \, \Delta s^4}{4(1-e^2)\,\nu\, r^3}$ =	−	0·0231	
− $\frac{15e^4 . \sin 4\lambda \, \Delta s^4}{32(1-e^2)^2\,\nu^2\, r^2}$ =	−	0·0003	
+ $\frac{e^2 . \cos 2\lambda \, \Delta s^5}{10(1-e^2)\,\nu\, r^4}$ =	+	0·0005	
Sum =		19423·6534 =	5°. 23'. 43"·6534

Computed & Compared by
Sreenath Sicdar
& Radhanauth Sen

Synopsis of the Foregoing Results.

Terrestrial Arc between Kalianpur and Sp. converted into seconds.

Value of the Arc in terms of the Sironj Base	1961155·435	Feet
Correction for difference between Sironj and Dehra Bases	+ 13·108	
Correction for error in the assumed height of the Sironj Base above the Sea Level	+ 2·295	
True value of the Arc	1961170·838	Log. 6·2925154268

Different Methods of Deduction	Col: Everest's Old Constants	Col: Everest's New Constants	Airy's constants	Bessel's Constants
By Using				
1st The Latitude of the Origin	5. 23. 43·513	5. 23. 42·430	5. 23. 43·232	5. 23. 42·653
2nd The Latitude of the Middle Point	43·514	42·431	43·234	42·655
3rd The Latitude of the Terminus	43·513	42·429	43·232	42·653
Mean Values	5. 23. 43·513	5. 23. 42·430	5. 23. 43·233	5. 23. 42·654

Compiled & Compared by
Sreenath Sicdar
& Radhanauth Sn

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Meridional distance from Kalianpur to u^{xxxix} Converted into Seconds.

Process No. 1

Consists in dividing the whole Meridional distance into eight parts and reducing each part by its own appropriate Radius of Curvature.

			Feet	Feet	Feet
1st	Kalianpur to u^{v} =		304969.223		
	Correction for difference between Sironj & Dehra Bases	η = +	0.821		
	Correction for Error in the height of the Sironj Base	δh = +	0.359	304969.903	3021.382
2nd	u^{v} to u^{xi} =		304239.600		
	Correction for difference between Sironj & Dehra Bases	η = +	0.960		
	Correction for error in the height of the Sironj Base	δh = +	0.956	304240.996	3013.829
3rd	u^{xii} to u^{xviii} =		271176.402		
	Correction for difference between Sironj & Dehra Bases	η = +	1.378		
	Correction for error in the height of the Sironj Base	δh = +	0.318	271178.098	2686.012
4th	u^{xviii} to u^{xxix} =		136999.588		
	Correction for difference between Sironj & Dehra Bases	η = +	0.336		
	Correction for error in the height of the Sironj Base	δh = +	0.160	137000.634	1356.882
5th	u^{xxix} to u^{xxxiii} =		340003.214		
	Correction for difference between Sironj & Dehra Bases	η = +	2.751		
	Correction for error in the height of the Sironj Base	δh = +	0.393	340006.363	3367.181
6th	u^{xxxiii} to u^{xxxiv} =		320212.458		
	Correction for difference between Sironj & Dehra Bases	η = +	3.309		
	Correction for error in the height of the Sironj Base	δh = +	0.375	320216.142	3170.779
7th	u^{xxxiv} to u^{xxxv} =		224044.572		
	Correction for difference between Sironj & Dehra Bases	η = +	3.520		
	Correction for error in the height of the Sironj Base	δh = +	0.332	234043.424	2312.303
8th	u^{xxxv} to u^{xxxix} =		310113.270		
	Correction for difference between Sironj & Dehra Bases	η = +	4.469		
	Correction for error in the height of the Sironj Base	δh = +	0.363	310123.102	3070.033
	Value of the whole Arc			2271733.572	22498.450 = 6.14.58.450

Computed & Compared by
Sreenath Sicdar
Gopeenauth Sen

Meridional distance from Kalianpur to u. Converted into seconds.

Process No. 2

Consists in making the reduction by the Radius of Curvature for Kalianpur in the following manner.

Latitude of Kalianpur = 24°.7'.12"

$r = 20818654{\cdot}63$ Feet..... Log. $r = 7{\cdot}3184526605$

$v = 20934537{\cdot}78$ Log. $v = 7{\cdot}3208634$

$$\frac{\Delta s}{r} = \quad + \quad 22508{\cdot}1307$$

$$- \quad \frac{3e^2 . \sin 2\lambda \, \Delta s^2}{4(1-e^2)\, v\, r} = \quad - \quad 9{\cdot}1312$$

$$- \quad \frac{e^2 . \cos 2\lambda \, \Delta s^3}{2(1-e^2)\, v\, r^2} = \quad - \quad 0{\cdot}5931$$

$$+ \quad \frac{e^4 . \sin^2 2\lambda \, \Delta s^3}{2(1-e^2)^2\, v^2\, r} = \quad + \quad 0{\cdot}0033$$

$$+ \quad \frac{e^2 . \sin 2\lambda \, \Delta s^4}{4(1-e^2)\, v\, r^3} = \quad + \quad 0{\cdot}0362$$

$$+ \quad \frac{15e^4 . \sin 4\lambda \, \Delta s^4}{32(1-e^2)^2\, v^2\, r^2} = \quad + \quad 0{\cdot}0006$$

$$+ \quad \frac{e^2 . \cos 2\lambda \, \Delta s^5}{10(1-e^2)\, v\, r^4} = \quad + \quad 0{\cdot}0014$$

Arc in seconds. = $22498{\cdot}4479 = 6^\circ.14'.58''{\cdot}448$

Computed & Compared by
Sremath Siedar
Gopeenauth Sen.

Meridional distance from Kalianpur to u^{xxxix}. Converted into seconds.

Process No. 3.

Consists in making the reduction by means of the Radius of Curvature for the Middle Point of the Arc.

Latitude of the Middle Point = 27°. 14'. 41"

r = 20827493·70 Feet ..... Log. r = 7·3186370117

ν = 20937500·12 Log. ν = 7·3209248269

$$\frac{\Delta s}{r} = + \quad 22498''\cdot5783$$

$$- \quad \frac{e^2 \cdot \cos 2\lambda \, \Delta s^3}{8(1-e^2)\,\nu\, r^2} = - \quad 0\cdot1292$$

$$+ \quad \frac{e^4 \cdot \sin^2 2\lambda \, \Delta s^3}{8(1-e^2)^2\,\nu^2\, r} = + \quad 0\cdot0010$$

$$+ \quad \frac{e^2 \cdot \cos 2\lambda \, \Delta s^5}{160(1-e^2)\,\nu\, r^4} = + \quad 0\cdot0001$$

Arc in Seconds = 22498·4502 = 6°. 14'. 58"·450

Synopsis of the Foregoing Results.

Meridional distance from Kalianpur to u^{xxxix}. Converted into Seconds.

Different Methods of Deduction	Arc Deduced
1st ..... By dividing the Meridional distance into eight parts and reducing each part by its own Radius of Curvature	6°. 14'. 58"·450
2d ..... By using the Latitude of the Origin	6. 14. 58·448
3d ..... By using the Mean Latitude	6. 14. 58·450
Mean Value	6. 14. 58·449

Compared & Compared by
Sreenath Sicdar
Gopeenauth Sen,

$\nu + \Delta\nu$ Expanded

$$\nu + \Delta\nu = \nu + \frac{d\nu}{d\lambda}\cdot\frac{\Delta\lambda}{1} + \frac{d^2\nu}{d\lambda^2}\cdot\frac{\Delta\lambda^2}{2} + \frac{d^3\nu}{d\lambda^3}\cdot\frac{\Delta\lambda^3}{2.3}$$

$$\nu = a(1-e^2.\sin^2\lambda)^{-\frac{1}{2}}$$

$$\frac{d\nu}{d\lambda} = \frac{1}{2}ae^2(1-e^2.\sin^2\lambda)^{-\frac{3}{2}}.\sin 2\lambda = \frac{\nu^3.e^2.\sin 2\lambda}{2a^2}$$

$$\frac{d^2\nu}{d\lambda^2} = \left\{\begin{array}{l}\frac{3}{4}ae^4(1-e^2.\sin^2\lambda)^{-\frac{5}{2}}.\sin^2 2\lambda \\ + ae^2(1-e^2.\sin^2\lambda)^{-\frac{3}{2}}.\cos 2\lambda\end{array}\right\} = \frac{3\nu^5.e^4.\sin^2 2\lambda}{4a^4} + \frac{\nu^3.e^2.\cos 2\lambda}{a^2}$$

$$\frac{d^3\nu}{d\lambda^3} = \left\{\begin{array}{l}\frac{15}{8}.ae^6(1-e^2.\sin^2\lambda)^{-\frac{7}{2}}.\sin^3 2\lambda \\ + \frac{9}{4}.ae^4(1-e^2.\sin^2\lambda)^{-\frac{5}{2}}.\sin 4\lambda \\ - 2ae^2(1-e^2.\sin^2\lambda)^{-\frac{3}{2}}.\sin 2\lambda\end{array}\right\} = \left\{\begin{array}{l}\frac{15\nu^7.e^6.\sin^3 2\lambda}{8a^6} + \frac{9\nu^5.e^4.\sin 4\lambda}{4a^4} \\ - \frac{2\nu^3.e^2.\sin 2\lambda}{a^2}\end{array}\right\}$$

Hence.

$$\nu + \Delta\nu = \nu + \frac{\nu^3.e^2.\sin 2\lambda.\Delta\lambda}{2a^2.\mathrm{Cosec}\,1''} + \frac{\nu^3.e^2.\cos 2\lambda.\Delta\lambda^2}{2a^2.\mathrm{Cosec}^2 1''} + \frac{3\nu^5.e^4.\sin^2 2\lambda.\Delta\lambda^2}{8a^4.\mathrm{Cosec}^2 1''}$$

$$- \frac{\nu^3.e^2.\sin^2 2\lambda\,\Delta\lambda^3}{8a^2.\mathrm{Cosec}^3 1''} + \frac{3\nu^5.e^4.\sin 4\lambda.\Delta\lambda^3}{8a^4.\mathrm{Cosec}^3 1''} + \frac{5\nu^7.e^6.\sin^3 2\lambda\,\Delta\lambda^3}{16a^6.\mathrm{Cosec}^3 1''}$$

$$- \frac{\nu^3.e^2.\sin 4\lambda.\Delta\lambda^4}{6a^2\,\mathrm{Cosec}^4 1''} + \ldots\ldots$$

$$- \frac{\nu^3.e^2.\cos 4\lambda\,\Delta\lambda^5}{120\,a^2\,\mathrm{Cosec}^5 1''} + \ldots\ldots$$

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Soonath Sircar
& Radhanauth Sen

Examples

Given the Normal for Latitude. 35°. Required the Normal for Latitude. 36.

$\lambda = 35°.$

$\Delta\lambda = 3600''.$

		Feet
$+ \frac{V^3 . e^2 . \sin 2\lambda . \Delta\lambda}{2 a^2 . \operatorname{Cosec} 1''}$	$=$	$+ 1142.6345$
$+ \frac{V^3 . e^2 . \cos 2\lambda \, \Delta\lambda^2}{2 a^2 . \operatorname{Cosec}^2 1''}$	$=$	$+ 7.2586$
$- \frac{V^3 . e^2 . \sin^2 2\lambda \, \Delta\lambda^3}{3 a^3 . \operatorname{Cosec}^3 1''}$	$=$	$- 0.2781$
$- \frac{V^3 . e^2 . \sin 4\lambda \, \Delta\lambda^4}{6 a^2 . \operatorname{Cosec}^4 1''}$	$=$	$- 0.0014$
$- \frac{V^3 . e^2 . \cos 4\lambda \, \Delta\lambda^5}{180 a^2 . \operatorname{Cosec}^4 1''}$	$=$	$- 0.0000$
$+ \frac{3 V^5 . e^2 . \sin^2 2\lambda \, \Delta\lambda^2}{8 a^4 . \operatorname{Cosec}^5 1''}$	$=$	$+ 10.0935$
$+ \frac{3 V^5 . e^4 . \sin 4\lambda \, \Delta\lambda^3}{8 a^4 . \operatorname{Cosec}^4 1''}$	$=$	$+ 0.0012$
$V + \Delta V =$		20946964.64
The same from independent computation		20946964.63
Discrepancy		$+ 0.01$

Computed & Compared by
Sreenath Sirdar
& Radhanauth Sen

Memo: regarding the computation of the Spherical Excess.-

The two formulæ in use in the Trigonometrical Survey for the computation of the Spherical Excess are as follows:-

1st When the two sides and the included angle are given,-

$$e'' = \frac{a.b.\sin C}{2r^2}.\operatorname{cosec} 1''.$$

2nd When one side and three angles are given,-

$$e'' = \frac{c^2.\sin A.\sin B}{2r^2.\sin C}.\operatorname{cosec} 1''$$

in which e" stands for the Spherical Excess in seconds. a, b, & c represent the sides of the triangle while the angles opposite thereto being respectively designated by A, B & C.- And lastly r denotes the normal to the Axis of the corresponding to the mean Latitude of the triangle.-

In the computation of the Spherical Excess, it is not necessary that the angles and sides required in the deduction should be known to any nicety.- If the former are taken to the nearest second and if the latter to the nearest foot the result deduced would be sufficiently accurate for all practical purposes.-

In taking the normal, the process generally adopted is as follows:- compute the mean latitude between the southernmost & northernmost points of the triangle, and the normal corresponding thereto is taken as the mean normal for that triangle.- Although this procedure is very rigorous, still so much exactitude is not demanded in practice; for a large variation may take place

in the normal without inducing any alteration in the Spherical Excess deduced.-

By a little computation it will be easily found that the mean Latitude of a Triangle for which the normal is taken may be out by 10°,- and that that difference would not produce a discrepancy of 0·01 in the Spherical Excess.—

It therefore follows that the Spherical Excesses of Triangles lying within ten degrees of Latitude may be computed by one constant Normal answering to the mean latitude between the northernmost and southernmost points of the Triangulation,- the latitudes of these limits being taken out with a pair of compasses from the plan.-

To illustrate this computation by an example, take the first triangle of the South Parasnath Series which is one of the largest triangles in the Trigonometrical Survey, and compute its Spherical Excess, 1st by using Latitude 22°-23·5; 2nd Latitude 31°-23·5 & Lastly the mean Latitude 26°-53·8.

Cheinpur to Silabani - in feet - Log. 5.3792193

Stations	Angles for computation	Sines	Deduced Sides
Cheinpur	46- 16- 17.476	9.8589122	
Silabani	47- 19- 18.121	9.8662886	
Gurgaburu	86- 24- 24.403	9.9991454	
	Cheinpur to Gurgaburu		Log. 5.2464625
	Silabani to Do.		Log 5.2389861

Spherical Excess deduced.-

	1st Computation with Lat: 22°-23'.5	2nd Computation with Lat: 31°-23'.5	3rd Computation with Lat: 26-53.8
Log. 2	0.3010300	0.3010300	0.3010300
Normals	7.3208319	7.3210140	7.3209177
	7.3208319	7.3210140	7.3209177
Sum	14.9426938	14.9430580	14.9428654
A.C	15.0573062	15.0569420	15.0571346
Logs. of two given sides	5.2464625	5.2464625	5.2464625
	5.2389861	5.2389681	5.2389681
Log. sine of inclined angle	T.9991454	T.9991454	T.9991454
Cosec	5.3144251	5.3144251	5.3144251
Sum	0.8563253	0.8559611	0.8561537
Spherical Excess	7.18	7.18	7.18

Expansion of $u = e^2 . \sin^2 \lambda$

Let u & λ vary together, the former becoming $u + \Delta u$, and the latter $\lambda + \Delta \lambda$. Hence by Taylor's Theorem we will have.

$$u + \Delta u = u + \frac{du}{d\lambda} \cdot \frac{\Delta\lambda}{1} + \frac{d^2u}{d\lambda^2} \cdot \frac{\Delta\lambda^2}{1.2} + \frac{d^3u}{d\lambda^3} \cdot \frac{\Delta\lambda^3}{1.2.3} + \ldots\ldots\ldots$$

$$\frac{du}{d\lambda} = e^2 . \sin 2\lambda$$

$$\frac{d^2u}{d\lambda^2} = 2e^2 . \cos 2\lambda$$

$$\frac{d^3u}{d\lambda^3} = -4e^2 . \sin 2\lambda$$

$$\frac{d^4u}{d\lambda^4} = -8e^2 . \cos 2\lambda$$

$$\frac{d^5u}{d\lambda^5} = 16e^2 . \sin 2\lambda$$

Hence

$$u + \Delta u = u + e^2 . \sin 2\lambda . \Delta\lambda . \sin 1'$$
$$+ e^2 . \cos 2\lambda . \Delta\lambda^2 . \sin^2 1'$$
$$- \tfrac{4}{3} e^2 . \sin 2\lambda . \Delta\lambda^3 . \sin^3 1'$$
$$- \tfrac{4}{3} e^2 . \cos 2\lambda . \Delta\lambda^4 . \sin^4 1'$$
$$+ \tfrac{4}{15} e^2 . \sin 2\lambda . \Delta\lambda^5 . \sin^5 1'$$
$$\ldots\ldots\ldots\ldots\ldots$$

As to the method of using this expansion for Tabulating $u = e^2 . \sin^2 \lambda$. for every 10" of latitude. Suppose λ to be any whole degree and substitute successively for $\Delta\lambda$. the following values after they are converted to seconds 10", 20", 30", 40", 50" the resulting quantities will be the required tabular values of u. for $\lambda + 10$, $\lambda + 20$ ... $\lambda + 50$ respectively.

That these tabular values may be computed as conveniently as possible, it is necessary to consider the following terms as constant

$+ e^2 . \sin 2\lambda . (10' \times 60) . \sin 1'' = a$

$+ e^2 . \cos 2\lambda . (10' \times 60)^2 . \sin^2 1'' = b$

$- \frac{1}{2} e^2 . \sin 2\lambda . (10' \times 60)^3 . \sin^3 1'' = c$

$- \frac{1}{4} e^2 . \cos 2\lambda . (10' \times 60)^4 . \sin^4 1'' = d$

$+ \frac{1}{10} e^2 . \sin 2\lambda (10' \times 60)^5 . \sin^5 1'' = e$

$\Delta\lambda = 0$

$u_0 = u$

$\Delta\lambda = 10'$

$u_1 = u + a + b + c + d + e$

$\Delta\lambda = 20'$

$u_2 = u + 2a + 4b + 8c + 16d + 32e$

$\Delta\lambda = 30'$

$u_3 = u + 3a + 9b + 27c + 81d + 243e$

$\Delta\lambda = 40'$

$u_4 = u + 4a + 16b + 64c + 256d + 1024e$

The first order of difference may be computed as follows.

$\Delta u_0 = u_1 - u_0 = a + b + c + d + e$

$\Delta u_1 = u_2 - u_1 = a + 3b + 7c + 15d + 31e$

$\Delta u_2 = u_3 - u_2 = a + 5b + 19c + 65d + 211e$

$\Delta u_3 = u_4 - u_3 = a + 7b + 37c + 175d + 781e$

The second order of difference may be deduced in the following manner

$\Delta^2 u_0 = \Delta u_1 - \Delta u_0 = 2b + 6c + 14d + 30e$

$\Delta^2 u_1 = \Delta u_2 - \Delta u_1 = 2b + 12c + 50d + 180e$

$\Delta^2 u_2 = \Delta u_3 - \Delta u_2 = 2b + 18c + 110d + 574e$

The deduction of the third order of difference may be performed in the following manner.

$\Delta^3 u_0 = \Delta^2 u_1 - \Delta^2 u_0 = 6c + 36d + 150e$

$\Delta^3 u_1 = \Delta^2 u_2 - \Delta^2 u_1 = 6c + 60d + 394e$

The computation of the fourth order of difference is effected in this way

$\Delta^4 u_0 = 24d + 240e$

Investigation of the formula for the reduction of the Barometer to 32°.

β = Observed Height of the Barometer.

C = Reduction of the same to 32°.

$\beta - C$ = Observed Height of Barometer reduced to 32°.

t = {Observed temperature of the mercury and of the Brass scale which are assumed to be equal.

m = {Expansion of mercury for 1° of Faht = .000100

b = Expansion of Brass for 1° of Faht = .0000106

32°. Standard temperature of the mercury.

62°. do do Brass.

Now the correction to Barometer for expansion of Mercury is this

$$+ (\beta - C)(t - 32^\circ)\, m$$

This is only one part of the correction, and the other part which arises from the dilation of the Brass scale is of the following form.

$$+ (\beta - C)(t - 62^\circ)\, b$$

Add the two terms together, and there will result the whole correction

$$- C = -(\beta - C)(t - 32^\circ)\, m + (\beta - C)(t - 62^\circ)\, b =$$

$$-(\beta - C)\{(t - 32^\circ)\, m - (t - 62^\circ)\, b\}$$

Therefore

$$C \times \{1 + (t - 32^\circ)\, m - (t - 62^\circ)\, b\} = \beta\{(t - 32^\circ)\, m - (t - 62^\circ)\, b\}$$

Hence

$$C = \beta \cdot \frac{(t - 32^\circ)\, m - (t - 62^\circ)\, b}{1 + (t - 32^\circ)\, m - (t - 62^\circ)\, b} = \beta \cdot \frac{(t - 32^\circ)\, m - (t - 62^\circ)\, b}{1 + (t - 32^\circ)\, m} \quad \text{nearly.}$$

Let $u = a^x$,

Then u & x becoming respectively $u + \Delta u$, & $x + \Delta x$, there will arise.

$$u + \Delta u = a^{x+\Delta x} = a^x \times a^{\Delta x} = u\left\{1 + A\Delta x + \frac{A^2 \Delta x^2}{2} + \frac{A^3 \Delta x^3}{2.3} + \ldots\ldots\right\}$$

Hence $\Delta u = u.A.\Delta x + \frac{u.A^2.\Delta x^2}{2} + \frac{u.A^3.\Delta x^3}{2.3} + \ldots\ldots\ldots$

If a is put $= 10$, then $A = 2.3025851$ which is the Hyperbolic or Napierian Logarithm of 10.

This value of Δu may be made use of in computing the natural number for a given logarithm in the following manner.

Suppose the given Logarithm to be $x + \Delta x$, the Tabular Logarithm next less being x. and u, the natural number corresponding thereto, then Δu is the correction to u consequent on Δx being added to x.

In the expanded value for Δx, the factors $A, \frac{A^2}{2}, \frac{A^3}{3}, \ldots$ are constants, the logarithm whereof may be computed off at once in the following manner.

L, (A) =	0·3622157	1st Constant
L, (A^2) =	0·7244314	
L, ($\frac{1}{2}$) =	7·6989700	
	0·4234014	2nd Constant
L, (A^3) =	1·0866471	
L, ($\frac{1}{6}$) =	7·2218487	
	0·3084958	3rd Constant

Example.

Example

Given Logarithm, $(x + \Delta x) = \cdot 109\ 1377\ 848$

Tabular D^{t} ..... $x = \cdot 109\ 1058\ 638$

$\Delta x = \quad 000\ 0319\ 210$

1st constant = 0·3622157 2nd constant = 0·42340.

x = 4·1091059 ..... 4·10911

$\log(\Delta x)$ = $\bar{5}$·5040769 ..... $\bar{5}$·50408 / $\bar{5}$·50408

$\bar{7}$·9753981 $\bar{5}$·54067

1st Term = ·944927 2nd Term 0,000035

1st Term ---------- 944927
2nd Term ---------- 35 } = Δw

Tabular natural number 12856 = w

sum ---------- 12856944962 = $w + \Delta w$

By interpolation ---------- 12856·9450 = d^{t}

By pursuing this computation it will be found that a logarithmic error of 000 0000 085 will produce the following discrepancies in the natural numbers in the different parts of the Table.

Δx = 000 0000 005

values of w	values of Δw
10000	·000012
20000	·000024
30000	·000036
40000	·000048
50000	·000058
60000	·000069
70000	·000081
80000	·000092
90000	·000104

Supposing the logarithms given in Vlacq's Table are true to unity in the 10th decimal, the natural numbers found ought to be as true as far as the 9th or 10th digit. This degree of accuracy is apparently not attainable in practice wherein the 8th digit is always found to be correct; but the 9th & 10th digits invariably wrong. For a proof of this vide the Table for $e^2 . \sin^2 n$.

Again take the following computation as a proof of Vlacq's Tables giving natural numbers true as far as the tenth figure.

Values of u	Log diff^{ce} for unity in the 5th figure of u.	Computed Values of Δu
10000 ——	0.000 0434 273 ——	1·000 0005
20000 ——	000 0217 141 ——	0·999 9962
30000 ——	000 0144 763 ——	1·000 0041
40000 ——	000 0108 573 ——	1·000 0067
50000 ——	000 0086 858 ——	0·999 9994
60000 ——	000 0072 382 ——	1·000 0028
70000 ——	000 0062 042 ——	1·000 0066
80000 ——	000 0054 286 ——	1·000 0007
90000 ——	000 0048 255 ——	1·000 0068

The computed values of Δu ought in every instance to have amounted to unity - but they do not do so, the discrepancies however (considering $u + \Delta u$ as one number) occur in the 11th & 12th places, showing that Vlacq's Tables are capable of furnishing natural numbers true to the 10th digit within an unit or so.

To Compute $\Delta u, \Delta^2 u, \Delta^3 u \ldots\ldots \Delta^b u$. for $u = \overline{x}^x$.

Now let $x + u$ become simultaneously $x + \Delta x + u + \Delta u$. we get.

$$u + \Delta u = u + \frac{du}{dx}\cdot\Delta x + \frac{d^2u}{dx^2}\cdot\frac{\Delta x^2}{2} + \frac{d^3u}{dx^3}\cdot\frac{\Delta x^3}{2.3} + \frac{d^4u}{dx^4}\cdot\frac{\Delta x^4}{2.3.4} + \frac{d^5u}{dx^5}\cdot\frac{\Delta x^5}{2.3.4.5} + \frac{d^6u}{dx^6}\cdot\frac{\Delta x^6}{2.3.4.5.6} + \ldots$$

For Δx, substitute nh. and we get.

$$u + \Delta u = u + an + bn^2 + cn^3 + dn^4 + en^5 + fn^6 + \ldots\ldots$$

in which, $a = \frac{du}{dx}h$ $b = \frac{d^2u}{dx^2}\cdot\frac{h^2}{2}$ $c = \frac{d^3u}{dx^3}\cdot\frac{h^3}{2.3}$ $d = \frac{d^4u}{dx^4}\cdot\frac{h^4}{2.3.4}$ $e = \frac{d^5u}{dx^5}\cdot\frac{h^5}{2.3.4.5}$

$$f = \frac{d^6u}{dx^6}\cdot\frac{h^6}{2.3.4.5.6} + \ldots\ldots$$

Now make n successivsly equal to $0, 1, 2, 3, 4, \ldots\ldots$ the corresponding values of u, will be as follows.

$u_0 = u$

$u_1 = u + a + b + c + d + e + f + \ldots\ldots$

$u_2 = u + 2a + 4b + 8c + 16d + 32e + 64f + \ldots\ldots$

$u_3 = u + 3a + 9b + 27c + 81d + 243e + 729f + \ldots\ldots$

$u_4 = u + 4a + 16b + 64c + 256d + 1024e + 4096f + \ldots\ldots$

$u_5 = u + 5a + 25b + 125c + 625d + 3125e + 15625f + \ldots\ldots$

$u_6 = u + 6a + 36b + 216c + 1296d + 7776e + 46656f + \ldots\ldots$

First order of difference computed.

$\Delta u_0 = u_1 - u_0 = a + b + c + d + e + f + \ldots\ldots$

$\Delta u_1 = u_2 - u_1 = a + 3b + 7c + 15d + 31e + 63f + \ldots\ldots$

$\Delta u_2 = u_3 - u_2 = a + 6b + 19c + 65d + 211e + 665f + \ldots\ldots$

$\Delta u_3 = u_4 - u_3 = a + 7b + 37c + 175d + 781e + 3367f + \ldots\ldots$

$\Delta u_4 = u_5 - u_4 = a + 9b + 61c + 369d + 2101e + 11529f + \ldots\ldots$

$\Delta u_5 = u_6 - u_5 = a + 11b + 91c + 671d + 4651e + 31031f + \ldots\ldots$

Second order of difference Computed

$\Delta^2 u_1 = \Delta u_1 - \Delta u_0 = 2b + 6c + 14d + 30e + 62f + \ldots\ldots$

$\Delta^2 u_2 = \Delta u_2 - \Delta u_1 = 2b + 12c + 50d + 180e + 602f + \ldots\ldots$

$\Delta^2 u_3 = \Delta u_3 - \Delta u_2 = 2b + 18c + 110d + 570e + 2702f + \ldots\ldots$

$\Delta^2 u_4 = \Delta u_4 - \Delta u_3 = 2b + 24c + 194d + 1320e + 8162f + \ldots\ldots$

$\Delta^2 u_5 = \Delta u_5 - \Delta u_4 = 2b + 30c + 302d + 2550e + 19502f + \ldots\ldots$

Third order of difference Computed

$\Delta^3 u_0 = \Delta^2 u_1 - \Delta^2 u_0 = 6c + 36d + 150e + 540f + \ldots\ldots$

$\Delta^3 u_1 = \Delta^2 u_2 - \Delta^2 u_1 = 6c + 60d + 390e + 2100f + \ldots\ldots$

$\Delta^3 u_2 = \Delta^2 u_3 - \Delta^2 u_2 = 6c + 84d + 750e + 5460f + \ldots\ldots$

$\Delta^3 u_3 = \Delta^2 u_4 - \Delta^2 u_3 = 6c + 108d + 1230e + 13340f + \ldots\ldots$

Fourth order of difference Computed

$\Delta^4 u_0 = \Delta^3 u_1 - \Delta^3 u_0 = 24d + 240e + 1560f + \ldots\ldots$

$\Delta^4 u_1 = \Delta^3 u_2 - \Delta^3 u_1 = 24d + 360e + 3360f + \ldots\ldots$

$\Delta^4 u_2 = \Delta^3 u_3 - \Delta^3 u_2 = 24d + 480e + 5880f + \ldots\ldots$

Fifth order of difference Computed

$\Delta^5 u_0 = \Delta^4 u_1 - \Delta^4 u_0 = 120e + 1800f + \ldots\ldots$

$\Delta^5 u_1 = \Delta^4 u_2 - \Delta^4 u_1 = 120e + 2520f + \ldots\ldots$

Sixth order of difference Computed

$\Delta^6 u_0 = \Delta^5 u_1 - \Delta^5 u_0 = 720f + \ldots\ldots$

Now take the first term in each order of difference and for $a, b, c, \ldots\ldots$ substitute their original values. The resulting series will be as follow

$\Delta u_0 = \frac{du}{dx}\cdot\frac{h}{1} + \frac{d^2u}{dx^2}\cdot\frac{h^2}{2} + \frac{d^3u}{dx^3}\cdot\frac{h^3}{6} + \frac{d^4u}{dx^4}\cdot\frac{h^4}{24} + \frac{d^5u}{dx^5}\cdot\frac{h^5}{120} + \frac{d^6u}{dx^6}\cdot\frac{h^6}{720} + \ldots\ldots$

$\Delta^2 u_1 = \frac{d^2u}{dx^2}\cdot\frac{h^2}{1} + \frac{d^3u}{dx^3}\cdot\frac{h^3}{1} + \frac{d^4u}{dx^4}\cdot\frac{7h^4}{12} + \frac{d^5u}{dx^5}\cdot\frac{h^5}{4} + \frac{d^6u}{dx^6}\cdot\frac{31h^6}{360} + \ldots\ldots$

$\Delta^3 u_2 = \frac{d^3u}{dx^3}\cdot\frac{h^3}{1} + \frac{d^4u}{dx^4}\cdot\frac{3h^4}{2} + \frac{d^5u}{dx^5}\cdot\frac{5h^5}{4} + \frac{d^6u}{dx^6}\cdot\frac{3h^6}{4} + \ldots\ldots$

$\Delta^4 u_3 = \frac{d^4u}{dx^4}\cdot\frac{h^4}{1} + \frac{d^5u}{dx^5}\cdot\frac{2h^5}{1} + \frac{d^6u}{dx^6}\cdot\frac{13h^6}{6} + \ldots\ldots$

$\Delta^5 u_4 = \frac{d^5u}{dx^5}\cdot\frac{h^5}{1} + \frac{d^6u}{dx^6}\cdot\frac{5h^6}{2} + \ldots\ldots$

$\Delta^6 u_5 = \frac{d^6u}{dx^6}\cdot\frac{h^6}{1} + \ldots\ldots$

The use of the foregoing expressions for computing a set of Tables may be explained in the following manner.

Suppose, $u = Fx$ is required to be tabulated,

Commencing from $x = a$ and advancing with the successive additions of w to a.

The first six Differential Coefficients of u, being computed, it is clear that, $\Delta u_0, \Delta^2 u_0 \ldots\ldots \Delta^6 u_0$ will become known.

In the expressions of $\Delta u_0, \Delta^2 u_0 \ldots\ldots \Delta^6 u_0$ a, being substituted for x, we will have the numerical values of these terms.

These numerical values will show what orders of difference are effective and what are non-effective. For instance, suppose the Table to consist of thirteen decimals, then if.

$\Delta u_0 = + 000\ 0108\ 437\ 98.0$

$\Delta^2 u_0 = + 000\ 0000\ 927\ 607$

$\Delta^3 u_0 = - 000\ 0000\ 003\ 690$

$\Delta^4 u_0 = + 000\ 0000\ 000\ 054$

it is evident that in a Table of this kind, it will not be necessary to go beyond the fourth order of difference.

Calling $\Delta^n u$ the last effective order of difference, compute $\Delta^n u$ for $x = a$, $x = a+h$, $x = a+2h \ldots\ldots$ and designate the deduced terms in the order in which they stand by. $\Delta^n u_0, \Delta^n u_1, \Delta^n u_2 \ldots\ldots$

$\Delta^n u_0, \Delta^n u_1, \Delta^n u_2 \ldots\ldots$ being known, the several differences of the $(n-1)$ order are as follows.

$\Delta^{n-1} u_0$ a previously computed term.

$\Delta^{n-1} u_1 = \Delta^{n-1} u_0 + \Delta^n u_0$

$\Delta^{n-1} u_2 = \Delta^{n-1} u_1 + \Delta^n u_1$

$\Delta^{n-1} u_3 = \Delta^{n-1} u_2 + \Delta^n u_2$

Similarly $\Delta^{n-1} u_0$ being known the differences of $(n-2)$ order may be deduced, and by proceeding in this manner from the differences

of one order to those of the next lower order, the tabular terms u_1, u_2, u_3 will eventually become determined.

The advantage attending the deduction of $u = \overline{\mathrm{E}}x$ by successive orders of differences as explained above arises from the circumstance of $\Delta^n u$ being of easier computation than $u = \overline{\mathrm{E}}x$.

To compute the area of a Spherical Triangle.

The rigorous expression for the area of a Spherical Triangle is this:

$$\tan \tfrac{1}{2}D = \frac{\tan \tfrac{1}{2}a \cdot \tan \tfrac{1}{2}b \cdot \sin C}{1 + \tan \tfrac{1}{2}a \cdot \tan \tfrac{1}{2}b \cdot \cos C}$$

in which D stands for the required area.

Now $\tfrac{1}{2}D = \tan \tfrac{1}{2}D - \tfrac{1}{3}\tan^3 \tfrac{1}{2}D$ nearly.

But by division and stopping at the 3rd term.

$$\tan \tfrac{1}{2}D = \begin{cases} \tan \tfrac{1}{2}a \cdot \tan \tfrac{1}{2}b \cdot \sin C \\ -\tfrac{1}{2}\tan^2 \tfrac{1}{2}a \cdot \tan^2 \tfrac{1}{2}b \cdot \sin 2C \\ +\tfrac{1}{2}\tan^3 \tfrac{1}{2}a \cdot \tan^3 \tfrac{1}{2}b \cdot \sin 2C \cdot \cos C \end{cases}$$

Also by cubing the first term of this expression.

$$\tan^3 \tfrac{1}{2}D = \tan^3 \tfrac{1}{2}a \cdot \tan^3 \tfrac{1}{2}b \cdot \sin^3 C.$$

Hence by combining, $\tan \tfrac{1}{2}D$, with $-\tfrac{1}{3}\tan^3 \tfrac{1}{2}D$ there will result.

$$\tfrac{1}{2}D = \begin{cases} \tan \tfrac{1}{2}a \cdot \tan \tfrac{1}{2}b \cdot \sin C \\ -\tfrac{1}{2}\tan^2 \tfrac{1}{2}a \cdot \tan^2 \tfrac{1}{2}b \cdot \sin 2C \\ +\tan^3 \tfrac{1}{2}a \cdot \tan^3 \tfrac{1}{2}b \cdot \sin C \cdot (1 - \tfrac{1}{3}\sin^2 C) \end{cases}$$

Suppose $\tfrac{1}{2}a = x$, and $\tfrac{1}{2}b = y$.

Now $\tan x = x + \tfrac{1}{3}x^3 + \tfrac{2}{15}x^5$.

$\tan y = y + \tfrac{1}{3}y^3 + \tfrac{2}{15}y^5$.

Hence

$$\tan x \cdot \tan y = \begin{cases} xy + \tfrac{1}{3}x^3y + \tfrac{2}{15}x^5y \\ + \tfrac{1}{3}xy^3 + \tfrac{1}{9}x^3y^3 \\ + \tfrac{2}{15}xy^5 \end{cases}$$

$$\tan^2 x \cdot \tan^2 y = \begin{cases} x^2y^2 + \tfrac{2}{3}x^4y^2 \\ + \tfrac{2}{3}x^2y^4 \end{cases}$$

$\tan^3 x \cdot \tan^3 y = x^3 \cdot y^3$.

Substituting these values in the expansion for $\tfrac{1}{2}D$ we will get as follows.

$$\tfrac{1}{2}D = \left\{\begin{array}{l} xy.\,Sin.C + \tfrac{1}{3}x^3y.\,Sin.C + \tfrac{2}{15}x^5y.\,Sin.C. \\ \qquad + \tfrac{1}{3}xy^3\,Sin\,C + \tfrac{1}{9}x^3y^3.\,Sin.C. \\ \qquad\qquad + \tfrac{2}{15}xy^5.\,Sin.C. \\ -\tfrac{1}{4}x^2y^2.\,Sin\,2C - \tfrac{1}{3}x^4y^2.\,Sin\,2C. \\ \qquad - \tfrac{1}{3}x^2y^4.\,Sin\,2C \\ \qquad + x^3y^3.\,Sin\,C\,(1-\tfrac{4}{3}Sin^2C). \end{array}\right. = 0$$

or

$$D = \left\{\begin{array}{l} 2xy.\,Sin.C. \\ +\tfrac{2}{3}x^3y.\,Sin\,C + \tfrac{2}{3}xy^3.\,Sin.C - x^2y^2.\,Sin\,2C. \\ +\tfrac{2}{15}x^5y.\,Sin\,C + \tfrac{2}{15}xy^5\,Sin\,C. \\ -\tfrac{2}{3}x^4y^2.\,Sin\,2C - \tfrac{2}{3}x^2y^4.\,Sin\,2C. \\ +\tfrac{4}{9}x^3y^3.\,Sin\,C\,(.5-6\,Sin^2C). \end{array}\right. = 0$$

in which $a \times b$ being restored and all the terms made homogeneous there will result.

$$D = \left\{\begin{array}{l} \tfrac{1}{2}ab.\,Sin\,C \\ \tfrac{1}{24r^2}.a^3b.\,Sin\,C + \tfrac{1}{24r^2}.ab^3.\,Sin\,C - \tfrac{1}{16r^2}.a^2b^2\,Sin\,2C. \\ +\tfrac{1}{240r^4}.a^5.b.\,Sin\,C + \tfrac{1}{240r^4}.a.b^5.\,Sin\,C. \\ -\tfrac{1}{96r^4}.a^4.b^2.\,Sin\,2C - \tfrac{1}{96r^4}.a^2.b^4.\,Sin\,2C. \\ +\tfrac{1}{144r^4}.a^3.b^3.\,Sin\,C\,(5-6\,Sin^2C.) \end{array}\right. = 0$$

which is the expression for the area of a Spherical Triangle.

Suppose it be required to compute the area of a square whose side is a. In this case it is evident that $a=b$. and that $C=\tfrac{1}{2}\pi$ $Sin\,C$ being $=1$ and $Sin\,2C=0$. These substitutions being made there will ari

$$D = \left\{\begin{array}{l} \tfrac{1}{2}a^2 + \frac{a^4}{24r^2} + \frac{a^4}{24r^2}. \\ +\frac{a^6}{240r^4} + \frac{a^6}{240r^4}. \\ -\frac{a^6}{144r^4}. \end{array}\right. =$$

But D is half the area of the square the double where of is.

$2D = a^2 + \frac{a^4}{6r^2} + \frac{a^6}{360r^4}$. which is the required expression for the superficial contents of the square.

The area of the square above mentioned may be more expeditiously found in the following manner.

$$\tan \tfrac{1}{2}D = \frac{\tan \frac{1}{2}a \cdot \tan \frac{1}{2}b \cdot \operatorname{Sin} C}{1 + \tan \frac{1}{2}a \cdot \tan \frac{1}{2}b \cdot \operatorname{Cos} C}$$

Now in a square. $a = b$, and $C = \frac{1}{2}\pi$. Hence $\operatorname{Sin} C = 1$. and $\operatorname{Cos} C = 0$.

These substitutions being made we will have.

$\tan \frac{1}{2}D = \tan^2 \frac{1}{2}a$.

Now $\frac{1}{2}D = \tan \frac{1}{2}D - \frac{1}{3}\tan^3 \frac{1}{2}D = \tan^2 \frac{1}{2}a - \frac{1}{3}\tan^6 \frac{1}{2}a$. nearly.

But.

$\tan \frac{1}{2}a = \frac{1}{2}a + \frac{1}{24}a^3 + \frac{1}{240}a^5 + \ldots\ldots\ldots$

$\tan^2 \frac{1}{2}a = \frac{1}{4}a^2 + \frac{1}{24}a^4 + \frac{17}{2880}a^6 + \ldots\ldots\ldots$

$\tan^6 \frac{1}{2}a = \frac{1}{64}a^6 + \ldots\ldots\ldots$

Hence.

$$\tfrac{1}{2}D = \begin{cases} \frac{1}{4}a^2 + \frac{1}{24}a^4 + \frac{17}{2880}a^6 + \ldots\ldots\ldots \\ \qquad - \frac{1}{192}a^6 + \ldots\ldots\ldots \end{cases}$$

and $D = \frac{1}{2}a^2 + \frac{1}{12}a^4 + \frac{1}{720}a^6 + \ldots\ldots\ldots$

Which last doubled becomes.

$2D = a^2 + \frac{1}{6}a^4 + \frac{1}{360}a^6 + \ldots\ldots\ldots$

Which is the expression for the required area of the given square.

Required to compute the Logarithm to a given natural number.

Let the given natural number be $= x + \Delta x$, *x being the part whose logarithm is found in Vlacq's Table;* Δx *being the remaining part for which the logarithmic correction is required to be computed*

Let $u = \mathrm{L}x$, *and also let u become* $u + \Delta u$, *when x becomes* $x + \Delta x$, *that is to say let* $u + \Delta u = \mathrm{L}(x + \Delta x)$

Now $\mathrm{L}(x + \Delta x) = \mathrm{L}x + \mathrm{L}(1 + \frac{\Delta x}{x}) = u + \Delta u$, *as supposed,*

Hence $\Delta u = m(\frac{\Delta x}{x} - \frac{\Delta x^2}{2x^2} + \frac{\Delta x^3}{3x^3} - \&c)$ *In this series the first two terms; + − being taken will give the logarithmic correction true to the* 10th *decimal and greater accuracy than this is not required for Vlacq's Table. The + − terms themselves may be computed logarithmically,* Δu *being computed and applied to u, will furnish the logarithm of* $(x + \Delta x)$

Example

Given natural number $x + \Delta x = 99783.43404387$ *in which* $x = 99783$. *of which the logarithmic* $u = 4.999\ 0565\ 370$.

	1st term		2d term
u	4.9990566		
A.C.	5.0009434	Doubled	10.002
$\Delta x = .43404387$. log.	T.6375337	"	T.275
$\mathrm{L}(m)$	T.6377843	$\mathrm{L}(\frac{m}{x})$	T.3367543
	6.2762614		12.614

The natural numbers of which sums are.

	+ 000 0018 891.3
	− 0.0
$\Delta u =$	+ 000 0018 891
$u =$	999 0565 570
$u + \Delta u =$	999 0584 461.

To ascertain the amount of accuracy attainable by this method, x may be successively taken as equal to 10000; 20000 30000 ..... 90000 and the logarithmic increment Δw, for Δx = unity, may be computed for these several values of x. This done the Δw may be applied in the order they stand to the Tabular logarithms of 10000, 20000 30000 ..... 90000 which will furnish the logarithms of 10001, 20001, 30001 ..... 90001. and these last logarithms may be tested by comparing them with their values as given in the Table. This comparison has actually been made. the result of which is that this mode of computing the proportional part to a logarithm occasionally entails an error of unity in the 10th decimal as may be seen by the following statement.

Natural Numbers	Logarithms as computed by the Formula.	Logarithms as found in the Table
10001	000 0434 273	000 0434 273
20001	301 0517 099*	301 0517 098
30001	477 1357 309*	477 1357 310
40001	602 0708 485*	602 0708 486
50001	698 9786 901	698 9786 901
60001	778 1584 886	778 1584 886
70001	845 1042 442	845 1042 442
80001	903 0954 156	903 0954 156
90001	954 2473 349	954 2473 349

The discrepant Logarithms are marked with an asterisk. It will be remembered that the cases given in the statement are extreme cases; for in practice Δx will be invariably less than unity in the fifth place of the natural number.

The method of computation therefore as shown in this paper being equal to these extreme cases, will likewise satisfy others, that are not so.

A

Formula for the Computation to the 15th Decimal.

Given the logarithm of x, it is required to compute the logarithm of $(x+h)$.

$$\mathrm{L}(x+h) = \mathrm{L}\,x + \mathrm{L}\left(\frac{x+h}{x}\right) = \mathrm{L}\,x + \mathrm{L}\left\{\frac{1+\frac{h}{2x+h}}{1-\frac{h}{2x+h}}\right\} =$$

$$\mathrm{L}\,x + \mathrm{L}\left(1+\frac{h}{2x+h}\right) - \mathrm{L}\left(1-\frac{h}{2x+h}\right) =$$

$$\mathrm{L}\,x + m\left\{\left(\frac{h}{2x+h}\right) - \frac{1}{2}\left(\frac{h}{2x+h}\right)^2 + \frac{1}{3}\left(\frac{h}{2x+h}\right)^3\right\}$$

$$+ m\left\{\left(\frac{h}{2x+h}\right) + \frac{1}{2}\left(\frac{h}{2x+h}\right)^2 + \frac{1}{3}\left(\frac{h}{2x+h}\right)^3\right\} =$$

$$\mathrm{L}\,x + 2m\left\{\left(\frac{h}{2x+h}\right) + \frac{1}{3}\left(\frac{h}{2x+h}\right)^3 + \frac{1}{5}\left(\frac{h}{2x+h}\right)^5\right\}$$

This series may be used in the following manner. Suppose any number to be given, make x equal to the first three digits, and take out $\mathrm{L}\,x$, from Hutton's Table of Logarithms to 20 decimals. This done, put h equal to the remaining digits, and then compute arithmetically the following terms:

$$\left(\frac{h}{2x+h}\right);\quad \frac{1}{3}\left(\frac{h}{2x+h}\right)^3;\quad \frac{1}{5}\left(\frac{h}{2x+h}\right)^5$$

Adding these terms and multiplying the sum by $2m$, the resulting product again being added to $\mathrm{L}\,x$, the final sum so derived will be the required numerical value of $\mathrm{L}(x+h)$.

When the logarithms to 15 decimals are required, it will not be necessary to compute more than the first two terms.

Logarithm of the Equatorial Radius Computed to the 15th D

Example.

Let. $x + h = 209\ 229\ 31{\cdot}8$ Feet Equatorial Radius.

$x = 209000000$ $\{ L\,x \} = {\cdot}32014\ 62861\ 14054$

$h = 229318$

$(2x + h) = 418229318$

$\left(\frac{h}{2x+h}\right) = {\cdot}00054\ 83068\ 50167$

$\left(\frac{h}{2x+h}\right)^2 = {\cdot}00000\ 03006\ 40402$

$\left(\frac{h}{2x+h}\right)^3 = {\cdot}00000\ 00001\ 64843$

$\left(\frac{h}{2x+h}\right) = {\cdot}00054\ 83068\ 50167$

$\frac{1}{3}\left(\frac{h}{2x+h}\right)^3 = 00000\ 00000\ 54948$

sum $= 00054\ 83069\ 05115$

Which multiplied by the modulus becomes

$00023\ 81266\ 63281$

which repeat .... $00023\ 81266\ 63281$

$L\ x$ ..... $32014\ 62861\ 11054$

required $L\ (x + h)$ ..... $32062\ 26394\ 37616$

the 15th D

Logarithm of the Polar Radius Computed to the 15th Decimal

Let $x+h$ = 20853374.58 Feet Polar Radius =

x = 20800000.00 Lx = .31806 33349 62762

h = 53374.58

$(2x+h)$ = 41653374.58

$\left(\frac{h}{2x+h}\right)$ = .00128 13986 99102

$\left(\frac{h}{2x+h}\right)^2$ = .00000 16419 82626

$\left(\frac{h}{2x+h}\right)^3$ = .00000 00021 04034

$\left(\frac{h}{2x+h}\right)$ = .00128 13986 99102

$\frac{1}{3}\left(\frac{h}{2x+h}\right)^3$ = .00000 00007 01345

sum = .00128 13994 00447

Which multiplied by the modulus becomes

.00055 65046 88728

which repeat ..00055 65046 88728

Lx 31806 33349 62762

required $L(x+h)$ ..31917 63443 40218

Comp^d Bell

Logarithm of the Sum of the Equatorial & Polar Radii computed to the 15th Dec

Let $x+h$ = 417 763 06.38 feet. Sum of the Equatorial & Polar Radii

x = 417 0000000.

h = 7630638 $\quad$ Lx = .62013 60549 73758

$(2x+h)$ = 8347630638

$\left(\frac{h}{2x+h}\right)$ = .00091 41082 45909

$\left(\frac{h}{2x+h}\right)^2$ = .00000 08355 93885

$\left(\frac{h}{2x+h}\right)^3$ = .00000 00007 63823

$\left(\frac{h}{2x+h}\right)$ = .00091 41082 45909

$\frac{1}{3}\left(\frac{h}{2x+h}\right)^3$ = .00000 00002 54608

Sum = .00091 41085 00517

Which multiplied by the modulus becomes

.00039 69922 77635

which repeat ..... .00039 69922 77635

Lx ..... .62013 60549 73758

required L$(x+h)$ .... .62093 00395 29028

15th De

rithm of the Difference between the Equatorial & Polar Radii Computed to the 15th Decimal

Radii

Let $x+h = 69557.22$ feet Difference between the Equatorial & Polar Radii

$x = 6950000$ $\quad$ Lx = .84198 48045 90114

$h =$ 57.22

$(2x+h) =$ 13905722

$\left(\frac{h}{2x+h}\right) =$ .00041 14852 86417

$\left(\frac{h}{2x+h}\right)^2 =$ .00000 01693 20141

$\left(\frac{h}{2x+h}\right)^3 =$ .00000 00000 69673

$\left(\frac{h}{2x+h}\right) =$ .00041 14852 86417

$\frac{1}{3}\left(\frac{h}{2x+h}\right)^3 =$ .00000 00000 23224

sum = .00041 14853 09641

Which multiplied by the modulus becomes

.00017 87057 99361

which repeat ..... .00017 87057 99361

Lx ..... .84198 48045 90114

required L$(x+h)$ .... .84234 22161 88836

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Northern Section Kalianpur-Kaliana of the Great Arc Series.

Meridional distance $\overset{IV}{p}$ to $\overset{XXXXX}{\mu}$ converted to arc, using Col. Everests old constants.

Latitude of $\overset{IV}{p}$	29 . 30 . 54"775	Approximate being deduced from the Latitude & Longitude computation from Station to Station
$\overset{XXXXX}{\mu}$	30 . 22 . 10:255	
Mean	29 . 56 . 32·515	
For Mean Latitude L.V	7·318 8083 187	
For Mean Latitude L y	7·320 9819 293	

	Feet
Terrestrial Arc. $\overset{IV}{p}$ to $\overset{XXXXX}{\mu}$ in terms of Sironj Base	310607·892
Correction for difference between Sironj & Dehra Dun Bases	+ 4·476
Correction for error in the height of Sironj Base	+ 0·366
True value of $\overset{IV}{p}$ to $\overset{XXXXX}{\mu}$ or ΔS	310612·734
And its Logarithm is	5·492 2192 563

Hence.

$\frac{\Delta S}{N}$ = + 3074"·9359

$\frac{-e^2 . \cos 2V . \Delta S^3}{8(1-e^2) . V . \nu^2}$ = −0·0003

Arc in Seconds 3074·9356 = 0° 51' 14"·936

Kalianpur to $\overset{IV}{p}$ in arc = 5. 23. 43·513

Kalianpur to $\overset{XXXXX}{\mu}$ in arc 6. 14. 58·449 } Identical

The same deduced direct 6. 14. 58·449

On former occasions the Meridional distances Kalianpur to $\overset{IV}{p}$ and Kalianpur to $\overset{XXXXX}{\mu}$ having been converted into arc, it was necessary to compare the results of the two reductions. This comparison not having been performed before, is now executed in the way detailed above viz. By first converting the meridional distance $\overset{IV}{p}$ to $\overset{XXXXX}{\mu}$ into arc and then adding thereto the arc comprized

between Kalianpur and ~~p~~ IV, the resulting sum is the arc between Kalianpur and ~~xxxx~~ II, which may be compared with its value deduced direct. This has accordingly been done, and the result as shewn above is satisfactory.

Computed & Compared by
Sreenath Siddar
&
Shib narain Dutt
12th December 1850

Statement containing the numerical elements required by Captain T. Renny Tailyour Astron.l Ass.t G.l Trig.l Survey, as per his letter No. 62. dated the 28th November 1850. to the Chief Computer's address.—

Extract from the original computation sheets executed in Col: Everest's time, showing the latitudes, longitudes and reciprocal azimuths of Dhoiwala and Banog stations.—

Stations	Latitudes	Longitudes	Reciprocal Azimuths
Dhoiwala	30° 7' 12.744"	78° 4' 32.354"	177° 26' 6.299"
Banog	30° 28' 37.518"	78° 3' 25.906"	357° 25' 32.797"

Note by the Chief Computer

The fundamental elements, on which these deductions are based, are as follows.—

Kalianpur ..... Latitude ..... 24° 7' 11.851"

Longitude ..... 77° 41' 47.18"

Azimuth of Surantal from Kalianpur ..... 190° 27' 6.205"

The distances used in the latitude and longitude computation of section Kalianpur-Kaliana are those corrected for the difference between the Sironj and Dehra Dun Bases, and not for the error in the height above the sea level of the former line.—Vide Col: Everest's printed book p. 224.

After the latitude and longitude deductions, of the Great Arc Series, were completed; the position of Kalianpur, as given above, was found to be wrong, the correct values whereof as then determined are as follows.—

Kalianpur ..... Latitude ..... 24° 7' 11.837"

Longitude ..... 77° 41' 44.748"

In consequence of this alteration in two of the fundamental elements, corrections −0.014" in latitude and −2.432" in longitude have been applied to all the deduced sites of the Great Arc Series to make them correspond to the new origin.—

The latitudes and longitudes of Dhoiwala & Banog stations, as registered in the Table are uncorrected elements.—

The last mentioned latitude of Kalianpur which has up to this moment been taken as a final result, has been derived from the astronomical observations of 1825.- But if in conjunction with these the like observations of 1839/40 and 1840/41 are made use of, the latitude of Kalianpur so deduced, will stand as follows.—

Kalianpur ..... Latitude ..... 24° 7' 11".262

This indicates the necessity of the application of a farther correction of -0".573 to the deduced latitudes of the Great Arc Series.—

If the corrections above mentioned are made use of, the final positions of Banog and Dhoiwala as can be determined at present, are as follows.—

Banog	Latitude	30° 28' 36".929
	Longitude	78° 3' 23".474
Dhoiwala	Latitude	30° 7' 12".155
	Longitude	78° 4' 29".922

Again the latitude and longitude of Banog as likewise the azimuth of Dhoiwala at that station may be derived from the meridional Triangles.- This deduction has accordingly been made; the results whereof are given below.—

Banog	Latitude	30° 28' 36".961
	Longitude	78° 3' 23".473
Azimuth of Dhoiwala from Banog ...		357° 25' 32".738

In making this computation Kalianpur is taken at latitude 24° 7' 11".262 and longitude 77° 41' 44".7 Also the whole meridional distance between Kalianpur and pe XXXIX has been reduced to the true sea level, and corrected for the difference between the Sirsij and Dehra Dun Bases.—

It will be seen that a discrepancy of 0".032 exists between the two values of the latitude of Banog derived respectively from the latitude and longitude computation, and the

Meridional Triangles; the value furnished by the latter being in excess.- This difference I think may partly be accounted for in the following manner.- In the computation of latitudes and longitudes of the stations of the Great Arc Series, the correction for error in the height of the Sirony Base was not applied to the distances as adverted to before.- This correction for the whole arc comprised between Kalianpur and μ[xxxix] amounts to 2·661 feet which converted into seconds become 0″·02661.- This therefore is the quantity whereby the latitude of Banog as deduced from the process of computing latitudes and longitudes from station to station, ought to fall short of that derived from the Meridional Triangles; and this in fact it does to within ·005 of a second.-

Again the two values of the longitude of Banog, obtained respectively from the latitude and longitude computation, and the Meridional Triangles, are, as they ought to be, identical.-

Finally as for the difference 0″·059 existing between the two azimuths of Dhoiwala derived from the two independent computations above mentioned it may perhaps be owing to the insufficient azimuth formula used in the latitude and longitude deductions.- on this subject, I beg to state, I am not competent to submit any opinion.-

The heights of Banog and Dhoiwala taking Col: Lambton's height of Damargida as the origin, are as follows.-

Height of Banog above the sea level ..... 7524·2 feet
" " Dhoiwala " " ..... 3039·8 "

The distance of Banog to Dhoiwala as derived from the recomputation of the triangles from the Dehra Dun Base is given below

Banog to Dhoiwala ..... feet 129918·569 Log. 5·1136712

In this deduction the length of the Dehra Dun Base is taken at 39183·87343 feet of which the Logarithm is 4·593 1073 650.-

Compared by
Radhanauth Sen
& Sreenath Sicdar

Surv. Generals Office
Calcutta, 17th Dec. 1850

S/d Radhanath Sickdhar
Chief Computer

Statement showing the [illegible] Latitude, Longitude and Azimuth formula, in a linear distance of 50 miles together with the errors produced by the reversed computation of the same formula.

Assumed Latitude of A = 32° [illegible]

	Direct computation. B deduced from A.	Reversed computation. A deduced from B.

Statement shewing the discrepancies engendered by limiting to three decimals the logarithms of the 3rd and 4th parts of the Latitude, Longitude and Azimuth formula, in a linear distance of 30 miles together with the errors produced by the reversed computation of the same formula.

Assumed Latitude of A = 34°

Assumed Latitude	Assumed Azimuth	Mode of deduction	Direct computation B deduced from A. Latitude	Difference of Longitude	Azimuth	Reversed computation. A deduced from B. Latitude	Error of Reversion	Difference of Longitude	Error in Reversion	Azimuth	Error in Reversion
0° 1' 0"		By the Method in use	34° 16' 28·4372	0° 0' 0·9152	180° 0' 59·4799	34° 59' 59·9997	·0003	0° 0' 0·9151	·0001	0° 1' 0·0000	....
		By the Method proposed	34° 16' 28·4373	0° 0' 0·9152	180° 0' 59·4799	34° 59' 59·9995	·0005	0° 0' 0·9151	·0001	0° 1' 0·0000	....
		Discrepancies	·0001	....	....	·0002		....		....	
89° 59' 0"		By the Method in use	34° 59' 47·7170	0° 52' 53·6195	269° 28' 39·7895	34° 59' 59·9990	·0010	0° 52' 53·6187	·0008	89° 58' 59·9998	·0002
		By the Method proposed	34° 59' 47·7170	0° 52' 53·6195	269° 28' 39·7895	34° 59' 59·9990	·0010	0° 52' 53·6187	·0008	89° 58' 59·9997	·0003
		Discrepancies.	....	....	....	....		....		·0001	
1° 0' 0"		By the Method in use	34° 16' 28·8819	0° 0' 54·9072	180° 59' 28·7910	35° 0' 0·0004	·0004	0° 0' 54·9072	·0001	0° 59' 59·9997	·0003
		By the Method proposed	34° 16' 28·8820	0° 0' 54·9073	180° 59' 28·7910	35° 0' 0·0002	·0002	0° 0' 54·9072	·0001	0° 59' 59·9997	·0003
		Discrepancies.	·0001	....	....	·0002		....		....	
89° 0' 0"		By the Method in use	34° 59' 2·9080	0° 52' 52·6563	268° 29' 40·5744	34° 59' 59·9996	·0004	0° 52' 52·6561	·0002	89° 0' 0·0001	·0001
		By the Method proposed	34° 59' 2·9080	0° 52' 52·6563	268° 29' 40·5745	34° 59' 59·9998	·0002	0° 52' 52·6561	·0002	89° 0' 0·0002	·0002
		Discrepancies.	....	....	·0001	....		....		·0001	
15° 0' 0"		By the Method in use	34° 17' 56·6682	0° 13' 34·5134	194° 52' 16·8946	35° 0' 0·0005	·0005	0° 13' 34·5108	·0026	14° 59' 59·9976	·0024
		By the Method proposed	34° 17' 56·6683	0° 13' 34·5133	194° 52' 16·8943	35° 0' 0·0006	·0006	0° 13' 34·5108	·0026	14° 59' 59·9975	·0025
		Discrepancies	·0001	·0001	·0001	·0001		....		·0001	
30° 0' 0"		By the Method in use	34° 22' 15·4077	0° 26' 14·8722	209° 45' 3·7683	35° 0' 0·0011	·0011	0° 26' 14·8699	·0023	29° 59' 59·9976	·0024
		By the Method proposed	34° 22' 15·4074	0° 26' 14·8725	209° 45' 3·7679	35° 0' 0·0007	·0007	0° 26' 14·8705	·0020	29° 59' 59·9976	·0024
		Discrepancies.	....	·0003	·0004	·0004		·0006		....	
45° 0' 0"		By the Method in use	34° 29' 7·6769	0° 37' 10·2653	224° 38' 48·9666	35° 0' 0·0007	·0007	0° 37' 10·2632	·0021	44° 59' 59·9976	·0024
		By the Method proposed	34° 29' 7·6770	0° 37' 10·2650	224° 38' 48·9666	35° 0' 0·0009	·0009	0° 37' 10·2631	·0019	44° 59' 59·9977	·0023
		Discrepancies.	·0001	·0003	·0001	·0002		·0001		·0001	
60° 0' 0"		By the Method in use	34° 38' 5·6789	0° 46' 36·4219	239° 33' 57·5791	35° 0' 0·0004	·0004	0° 46' 36·4222	·0003	60° 0' 0·0005	·0005
		By the Method proposed	34° 38' 5·6792	0° 46' 36·4217	239° 33' 57·5792	35° 0' 0·0006	·0006	0° 46' 36·4224	·0007	60° 0' 0·0007	·0007
		Discrepancies	·0003	·0002	·0001	·0002		·0002		·0002	
75° 0' 0"		By the Method in use	34° 48' 33·3975	0° 50' 58·5190	254° 30' 49·8539	35° 0' 0·0000	....	0° 50' 58·5208	·0018	75° 0' 0·0021	·0021
		By the Method proposed	34° 48' 33·3976	0° 50' 58·5189	254° 30' 49·8540	35° 0' 0·0000	....	0° 50' 58·5208	·0019	75° 0' 0·0022	·0022
		Discrepancies	·0001	·0001	·0001	....		....		·0001	

Compared by
Hiren Coomar Mittra
Hiren Benerji

Statement shewing the discrepancies engendered by limiting to three decimals the logarithms of the 3rd and 4th parts of the Lat. rev. Longitude and Azimuth Formulæ in a linear distance of 100 miles together with the errors produced by the like reversed computation of the same formulæ.

Assumed	Mode of deduction	Direct computation, B deduced from A			Reversed computation, A deduced from B.					
		Latitude	Difference of Longitude	Azimuth	Latitude	Error of Reversion	Difference of Longitude	Error of Reversion	Azimuth	Error of Reversion
30. 0. 0	By the Method in use	33. 32. 56·5640	0. 0. 1·8148	180. 0. 58·9779	35. 0. 0·0008	·0008	0. 0. 1·8147	·0001	0. 0. 59·9999	·0001
	By the Method proposed	33. 32. 56·5646	0. 0. 1·8148	180. 0. 58·9779	35. 0. 0·0013	·0013	0. 0. 1·8147	·0001	0. 0. 59·9999	·0001
	Discrepancies	·0006	....	....	·0005		....		....	
34. 59. 59·9873	By the Method in use	34. 59. 12·3876	1. 45. 46·7281	268. 58. 20·0708	34. 59. 59·9873	·0127	1. 45. 46·7282	·0001	89. 59. 0·0010	·0010
	By the Method proposed	34. 59. 12·3876	1. 45. 46·7282	268. 58. 20·0708	34. 59. 59·9873	·0127	1. 45. 46·7272	·0011	89. 58. 59·9998	·0002
	Discrepancies	....	·0001	....	....		·0011		·0012	
0. 0. 0	By the Method in use	33. 32. 57·3470	0. 1. 48·8808	180. 58. 58·6771	35. 0. 0·0007	·0007	0. 1. 48·8779	·0029	0. 59. 59·9972	·0028
	By the Method proposed	33. 32. 57·3474	0. 1. 48·8807	180. 58. 58·6770	35. 0. 0·0008	·0008	0. 1. 48·8779	·0028	0. 59. 59·9971	·0029
	Discrepancies	·0004	·0001	·0001	·0001		....		·0001	
30. 0. 0	By the Method in use	34. 57. 42·7942	1. 45. 48·8400	267. 59. 22·3556	34. 59. 59·9887	·0113	1. 45. 48·8430	·0030	89. 0. 0·0019	·0019
	By the Method proposed	34. 57. 42·7943	1. 45. 48·8397	267. 59. 22·3561	34. 59. 59·9888	·0112	1. 45. 48·8442	·0045	89. 0. 0·0039	·0039
	Discrepancies	·0001	·0003	·0005	·0001		·0012		·0020	
15. 0. 0	By the Method in use	33. 35. 51·5700	0. 26. 56·6154	194. 44. 49·5054	35. 0. 0·0040	·0040	0. 26. 56·5796	·0359	14. 59. 59·9630	·0370
	By the Method proposed	33. 35. 51·5704	0. 26. 56·6149	194. 44. 49·5052	35. 0. 0·0029	·0029	0. 26. 56·5823	·0326	14. 59. 59·9665	·0335
	Discrepancies	·0004	·0005	·0002	·0011		·0028		·0035	
30. 0. 0	By the Method in use	33. 44. 25·2165	0. 52. 6·3603	209. 30. 34·9118	35. 0. 0·0127	·0127	0. 52. 6·3217	·0386	29. 59. 59·9594	·0406
	By the Method proposed	33. 44. 25·2165	0. 52. 6·3618	209. 30. 34·9092	35. 0. 0·0118	·0118	0. 52. 6·3222	·0396	29. 59. 59·9573	·0427
	Discrepancies	....	·0015	·0026	·0009		·0005		·0021	
45. 0. 0	By the Method in use	33. 58. 4·0380	1. 13. 53·2273	224. 18. 9·8457	35. 0. 0·0139	·0139	1. 13. 53·2148	·0125	44. 59. 59·9849	·0151
	By the Method proposed	33. 58. 4·0381	1. 13. 53·2252	224. 18. 9·8468	35. 0. 0·0140	·0140	1. 13. 53·2132	·0100	44. 59. 59·9869	·0131
	Discrepancies	·0001	·0021	·0011	·0001		·0004		·0020	
60. 0. 0	By the Method in use	34. 15. 54·3797	1. 30. 48·8063	239. 8. 23·1981	35. 0. 0·0080	·0080	1. 30. 48·8238	·0185	60. 0. 0·0166	·0166
	By the Method proposed	34. 15. 54·3813	1. 30. 48·8043	239. 8. 23·1989	35. 0. 0·0087	·0087	1. 30. 48·8233	·0190	60. 0. 0·0167	·0167
	Discrepancies	·0016	·0010	·0008	·0007		·0005		·0001	
75. 0. 0	By the Method in use	34. 36. 45·5198	1. 41. 42·7495	254. 1. 56·8358	34. 59. 59·9961	·0039	1. 41. 42·7718	·0223	75. 0. 0·0220	·0220
	By the Method proposed	34. 36. 45·5197	1. 41. 42·7490	254. 1. 56·8364	34. 59. 59·9960	·0040	1. 41. 42·7726	·0236	75. 0. 0·0232	·0232
	Discrepancies	·0004	·0006	·0006	·0001		·0008		·0012	

Surrebunar Mittra & Joynarain Banerji

On Minimum Squares.

Suppose $m', m'', m'''.....m^{(n)}$, to be the measured values of a certain magnitude, all differing from one another, n being their number. It is required to find the true value of that magnitude.

Let A be this true value, then it is evident that

$(m'-A)$ ... is the error of m'.

$(m''-A)$

$(m'''-A)$

$(m^{(n)}-A)$

Now take the sum of the squares of these errors, and put it equal to 2Ω, thus

$$(m'-A)^2 + (m''-A)^2 + (m'''-A)^2 ..... + (m^{(n)}-A)^2 = 2\Omega$$

Differentiating Ω with respect to A, there will result.

$$(m'-A) + (m''-A) + (m'''-A) ..... + (m^{(n)}-A) = -\frac{d.\Omega}{dA}.$$

Now that 2Ω may be a minimum, it is necessary that $\frac{d\Omega}{dA}$ should be equivalent to nothing. Making therefore $\frac{d\Omega}{dA} = 0$, there will arise

$$A = \frac{m' + m'' + m''' ..... + m^{(n)}}{n}.$$

That is to say the average of the observed values of a magnitude, is the true value of that magnitude, according to the doctrine of the minimum squares.

In the preceding investigation the measured values $m', m'', m''' ..... m^{(n)}$ are supposed to be of the same weight or entitled to the same amount of confidence. In practice this condition may or may not be fulfilled. Suppose it is not fulfilled;

and let $\dot{w}$ be the weight of $\dot{m}$, $\ddot{w}$ that of $\ddot{m}$, $\dddot{w}$ that of $\dddot{m}$ ..... and lastly $^{(n)}w$ that of $^{(n)}m$ .....

Then calling as before A the true value, take the square of the error of each measured value and multiply it by the weight appertaining thereto. Form these products into a sum and put it equal to 2a in this way.

$$\dot{w}(\dot{m}-A)^2 + \ddot{w}(\ddot{m}-A)^2 + \dddot{w}(\dddot{m}-A)^2 \ldots\ldots + {}^{(n)}w({}^{(n)}m-A)^2 = 2a$$

The differential whereof with respect to A is as follows.

$$\dot{w}(\dot{m}-A) + \ddot{w}(\ddot{m}-A) + \dddot{w}(\dddot{m}-A) \ldots\ldots + {}^{(n)}w({}^{(n)}m-A) = -\frac{da}{dA}.$$

To make 2a a minimum, put $\frac{da}{dA}$ equal to nothing and there results.

$$A = \frac{\dot{m}\dot{w} + \ddot{w}\ddot{m} + \dddot{w}\dddot{m} \ldots\ldots + {}^{(n)}w\,{}^{(n)}m}{\dot{w} + \ddot{w} + \dddot{w} \ldots\ldots + {}^{(n)}w}$$

This expression may be stated in words thus; Take the sum of the products of the measured values, each multiplied by its own weight and divide it by the sum of the weights, the resulting quotient is the true value of the magnitude measured agreeably to the doctrine of Minimum Squares

Suppose m, $\dot{m}$ and $\ddot{m}$ to be the measured values of the three angles of a Geodetic Triangle, w, $\dot{w}$, $\ddot{w}$ being their respective weights. Now let A be the true value of m. B that of $\dot{m}$ and C that of $\ddot{m}$. It follows that.

$m - A = \alpha$ is the error of m.

$\dot{m} - B = \beta$ Do $\dot{m}$

$\ddot{m} - C = \gamma$ Do $\ddot{m}$

Results of the Reduction of the Dehra Dun Trilateral according to the Theorem of the Minimum Squares.-

Stations	Probabilities	Observed Angles	Corrections	Spherical Excess
Banog	0·310	31-41-33·768	−0·31323	0·635
East End Base	0·358	76-26-38·094	−0·37156	
West End Base	0·325	71-41-49·943	−0·48522	
East End Base	0·428	3-4-3·489	+0·38500	0·948
Amsot	0·292	2-4-34·713	−0·04373	
West End Base	0·454	174-51-21·491	+0·01374	
Amsot	0·377	37-30-0·327	+0·90181	0·906
Banog	0·226	29-3-10·115	+0·52470	
West End Base	0·246	113-26-48·666	+0·47149	

Triangular Equations Satisfied

Triangle No. 1.

Banog	31 - 41 - 33'46477
E. End Base	76 - 36 - 37'72244
W. End Base	71 - 41 - 49'45778
	180 - 0 - 0'63499
Computed Spherical Excess	π + 0'63500
Error	0'00001

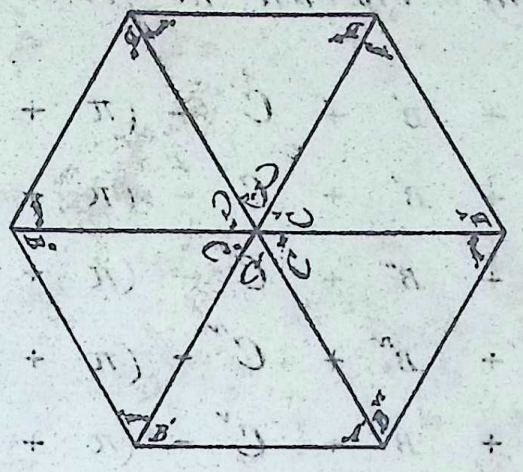
Triangle No. 2.

E. End Base	
Amsot	
W. End Base	
Computed Spherical Excess	
Error	

Triangle No. 3.

Amsot	
Banog	
W. End Base	
Computed Spherical Excess	
Error	

Formula for the Reduction of a Polygon agreeably to the Method of Minimum Squares



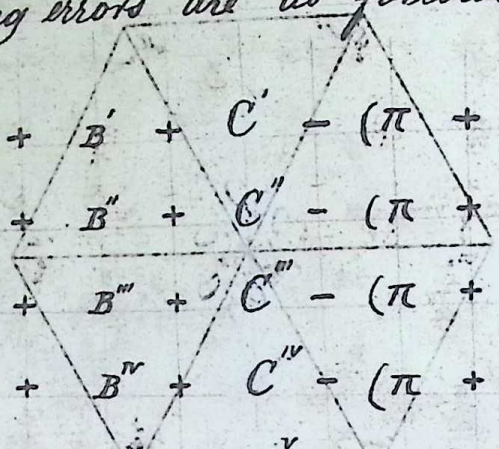
Explanation of the Symbols.

Triangles	Observed angles	Probabilities	Sum of the Probabilities	Spherical Excess	Triangular Error	Spherical Angles
A'	A'	a'				$A' + \delta A'$
B'	B'	b'	S'	e'	ε'	$B' + \delta B'$
C'	C'	c'				$C' + \delta C'$
A''	A''	a''				$A'' + \delta A''$
B''	B''	b''	S''	e''	ε''	$B'' + \delta B''$
C''	C''	c''				$C'' + \delta C''$
A'''	A'''	a'''				$A''' + \delta A'''$
B'''	B'''	b'''	S'''	e'''	ε'''	$B''' + \delta B'''$
C'''	C'''	c'''				$C''' + \delta C'''$
A^{IV}	A^{IV}	a^{IV}				$A^{IV} + \delta A^{IV}$
B^{IV}	B^{IV}	b^{IV}	S^{IV}	e^{IV}	ε^{IV}	$B^{IV} + \delta B^{IV}$
C^{IV}	C^{IV}	c^{IV}				$C^{IV} + \delta C^{IV}$
A^{V}	A^{V}	a^{V}				$A^{V} + \delta A^{V}$
B^{V}	B^{V}	b^{V}	S^{V}	e^{V}	ε^{V}	$B^{V} + \delta B^{V}$
C^{V}	C^{V}	c^{V}				$C^{V} + \delta C^{V}$
A^{VI}	A^{VI}	a^{VI}				$A^{VI} + \delta A^{VI}$
B^{VI}	B^{VI}	b^{VI}	S^{VI}	e^{VI}	ε^{VI}	$B^{VI} + \delta B^{VI}$
C^{VI}	C^{VI}	c^{VI}				$C^{VI} + \delta C^{VI}$

Error of the Central Equation $= \varepsilon^{VII}$

Error of the Reentering Equation $= \varepsilon^{VIII}$

The Algebraical values of the Triangular Central and Reentering errors are as follows. —



$$\varepsilon' = A' + B' + C' - (\pi + e')$$

$$\varepsilon'' = A'' + B'' + C'' - (\pi + e'')$$

$$\varepsilon''' = A''' + B''' + C''' - (\pi + e''')$$

$$\varepsilon^{IV} = A^{IV} + B^{IV} + C^{IV} - (\pi + e^{IV})$$

$$\varepsilon^{V} = A^{V} + B^{V} + C^{V} - (\pi + e^{V})$$

$$\varepsilon^{VI} = A^{VI} + B^{VI} + C^{VI} - (\pi + e^{VI})$$

$$\varepsilon''' - C' + C'' + C''' + C^{IV} + C^{V} + C^{VI} - 2\pi$$

$$\varepsilon^{VIII} + \left\{ \begin{array}{l} (L\sin A' + L\sin A'' + L\sin A''' + L\sin A^{IV} + L\sin A^{V} + L\sin A^{VI}) \\ -(L\sin B' + L\sin B'' + L\sin B''' + L\sin B^{IV} + L\sin B^{V} + L\sin B^{VI}) \end{array} \right\}$$

In computing these expressions arithmetically the operations of addition and subtraction as indicated by the of + and — should be performed and proper signs affixed to the resulting errors. —

From the errors deduced in this way the spherical angular corrections $\delta A'$, $\delta B'$, $\delta C'$ ..... are derived, the algebraical values whereof are given below. —

Observations on the eclipse of the sun which occurred on the 18th September 1857. Civil Time –

Prior to the eclipse of the sun McCabe Chronometer No 472 was compared with Thawaits and found to indicate as follows.

17 – Sept. Astronomical Time –

	h m s	
McCabe No 472	2 - 24 - 49 +	54
Thawaits ...	10 - 58 - 20 +	25
Difference	3 - 26 - 29	34

Objects Observed	Time Sidereal McCabe No 472	Corresponding Mean Time	Remarks
1st Contact of the Moon with the Sun	h m s 3 - 3 - 45	h m s 21 - 6 57.8	Beginning of the eclipse
Moon's Contact with 1st spot on the Sun: Lower edge Penumbra	3 12 12	21 15 24.8	
Lower edge Umbra ...	3 12 37	21 - 15 49.8	
Upper edge Umbra ..	3 13 10	21 - 16 22.8	
Upper edge Penumbra	3 13 36	21 - 16 48.8	
Moon's Contact with 2nd Spot on the Sun --	3 - 20 - 23	21 - 23 35.8	Crack spot centre.
Moon's Contact with 3rd spot on the left edge of the Sun -----	3 52 2	21 - 55 14.8	Crack spot & very faint
Moon's Contact with 4th Spot on the Sun ...	4 6 27.5	22 - 9 40.3	Crack spot centre.
Moon's Contact with 5th Spot on the Sun ...	4 8 23	22 - 11 35.8	Crack spot centre.

Objects Observed	Time by McCabe No 472	Corresponding Mean Time	Remarks
	h m s	h m s	
Venus visible to the naked eye	4-26-0	22-29 13	These are approximate Times
Greatest Obscuration	4-38-9	22-41-22	
Emersion 1st Spot	4-47-0	22-50 13	
2nd Spot	4-56-24	22-59-37	
3rd Spot	5 2-0	23-5 13	
4th Spot	5 35-0	23-38 13	
5th Spot	5-36-42	23-39 55	
Emersion of the Sun from the Moon's Shadow	Obscured by the clouds but occurred about 6h 18m 54s	24-22-7	

After the eclipse of the sun McCabe [illegible] No. 472 was compared with Thwaites and found to indicate as follows.

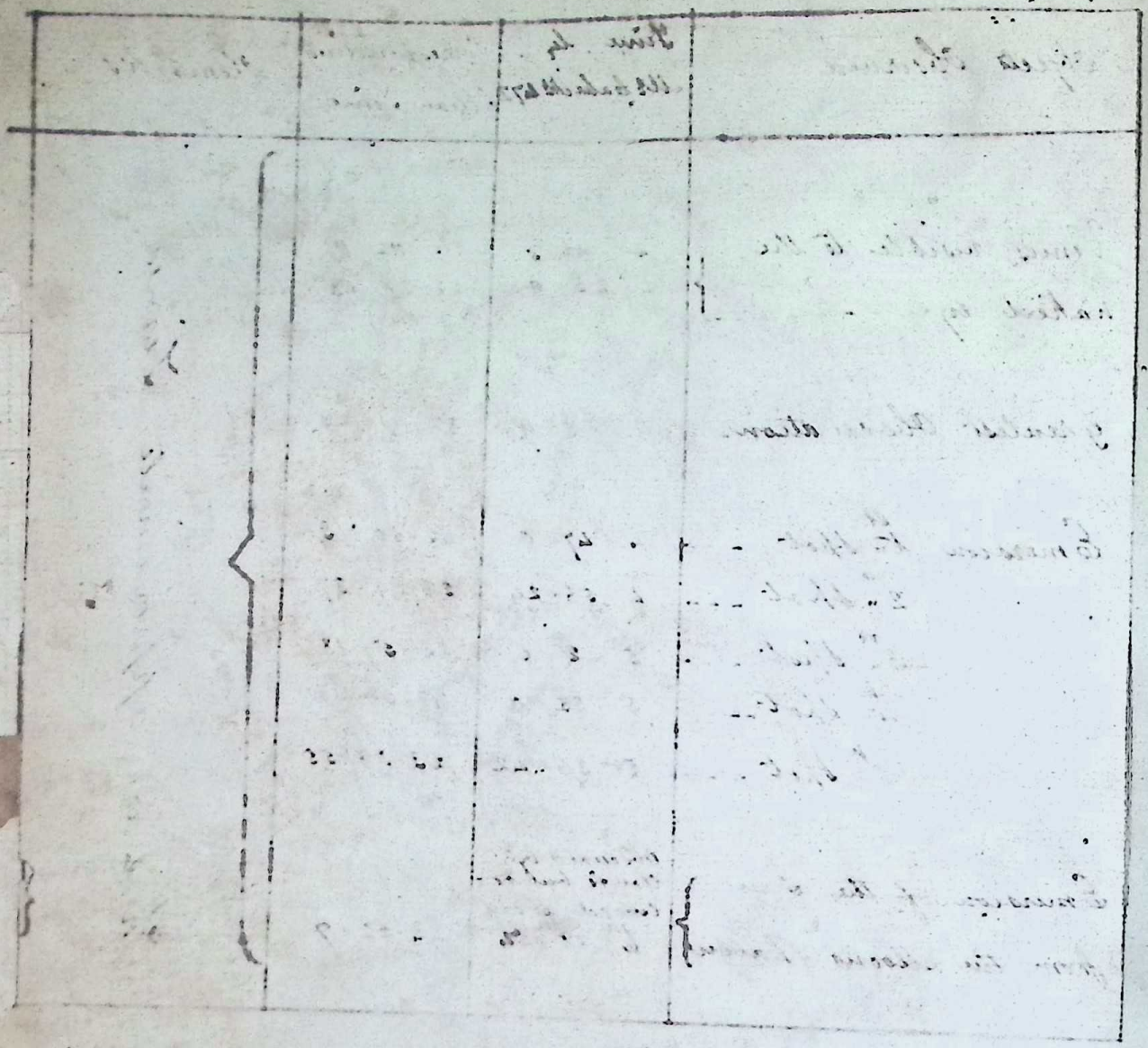
	h m s
McCabe No 472	6-21-2
Thwaites	2-54-30
Difference	3-26-32

118th Lehlentz [illegible]

During the eclipse as the sun and Moon were passing over the Meridian the following observations of the latter were taken

		Time by Thwaites	Corresponding Mean Time
Moon's 1st Limb	3rd Wire	2h 24m 51.0	23h 54m 36.2
	4th Wire	2-25-7.5	23-54 52.7

Computed & Compared by
Sreenath Sicdar & Panch Kori Gupta



Memo: regarding the Computation of $\Delta\lambda$ in the Latitude Longitude and Azimuth formula

The following Table shews the variations which would be induced in the term $\Delta\lambda$ corresponding to an alteration of 20″ in the given Azimuth upon a distance of 50 miles. –

Given Azimuth. –	Value of $\Delta\lambda$	Variation in $\Delta\lambda$
89° 59′ 50″	0° 19′ 22″·5359	0″·0022
90° 0′ 10″	0° 19′ 22″·5381	
80° 0′ 0″	0° 18′ 56″·6513	0″·0217
80° 0′ 20″	0° 18′ 56″·5720	
70° 0′ 0″	0° 18′ 0″·8652	0″·0404
70° 0′ 20″	0° 18′ 0″·9056	
60° 0′ 0″	0° 16′ 33″·0115	0″·0574
60° 0′ 20″	0° 16′ 33″·0689	
50° 0′ 0″	0° 14′ 35″·8844	0″·0725
50° 0′ 20″	0° 14′ 35″·9569	
40° 0′ 0″	0° 12′ 13″·1581	0″·0856
40° 0′ 20″	0° 12′ 13″·2437	
30° 0′ 0″	0° 9′ 29″·1655	0″·0959
30° 0′ 20″	0° 9′ 29″·2614	
20° 0′ 0″	0° 6′ 28″·7635	0″·1036
20° 0′ 20″	0° 6′ 28″·8671	
10° 0′ 0″	0° 3′ 17″·2039	0″·1082
10° 0′ 20″	0° 3′ 17″·3121	

Compared by
Sreenath Sircar

Examination of δZ by the Approximate Formula

Date	Time from Max Elongation	Values of δZ Rigorous	Values of δZ Approx.^t	Difference	Date	Time from Max Elongation	Values of δZ Rigorous	Values of δZ Approx.^t	Difference
	h m s	"	"	"		h m s	"	"	"
E. E. of α Ursæ Minoris taken at Pota on the 2nd April 1846 Aftermidnight	0 30 27	53.61	52.49	0.12	W. E. of α Ursæ Minoris taken at Pota on the 3rd April 1846 Afternoon	0 6 28	2.43	2.37	0.06
	0 27 20	43.22	42.28	0.94		0 4 48	1.34	1.30	0.04
	0 21 14	26.10	25.51	0.59		0 1 16	0.09	0.09	0.00
	0 18 33	19.93	19.48	0.45		0 2 35	0.39	0.38	0.01
	0 9 15	4.85	4.74	0.11		0 7 29	3.25	3.17	0.08
	0 6 59	2.83	2.77	0.06		0 9 37	5.36	5.24	0.12
	0 1 14	0.09	0.09	0.00		0 15 42	14.28	13.86	0.42
	0 5 23	1.68	1.64	0.04		0 17 21	17.44	17.04	0.40
E. E. of α Ursæ Minoris taken at Pota on the 3 April 1846 Aftermidnight	0 33 39	65.43	63.96	1.47	W. E. of α Ursæ Minoris taken at Pota on the 4 April 1846 Afternoon	0 10 28	6.36	6.21	0.16
	0 30 12	52.72	51.65	1.07		0 9 0	4.70	4.59	0.11
	0 22 40	29.73	29.07	0.66		0 1 57	0.22	0.22	0.00
	0 19 26	21.86	21.37	0.49		0 0 30	0.01	0.01	0.00
	0 9 18	5.01	4.90	0.11		0 5 53	2.01	1.96	0.05
	0 3 5	0.55	0.54	0.01		0 7 42	3.44	3.36	0.08
	0 3 19	0.64	0.62	0.02		0 13 52	11.15	10.89	0.26
	0 9 5	4.80	4.67	0.13		0 15 38	14.16	13.83	0.33

Examination of δz by the Approximate Formulæ

Date	Time from Max: Elongation	Values of δz Rigorous	Values of δz Approx:	Difference
E. E. of α Ursæ Minoris taken at Pola on 4th April 1846 Afternoon	h m s	"	"	"
	0 28 53	48·20	47·22	0·98
	0 27 4	42·34	41·48	0·86
	0 19 29	21·95	21·48	0·47
	0 16 55	16·55	16·21	0·34
	0 36 56	0·89	0·87	0·02
	0 2 24	0·33	0·33	0·00
	0 6 3	2·14	2·07	0·07
	0 8 4	3·79	3·69	0·10
E. E. of α Ursæ Minoris taken at Pola on the 6th April 1846 Afternoon	0 12 37	10·00	8·65	1·35
	0 10 2	6·86	5·70	0·16
	0 5 2	1·48	1·43	0·06
	0 1 49	0·19	0·19	0·00
	0 4 10	1·00	0·98	0·02
	0 5 53	2·00	1·96	0·04
	0 12 8	8·52	8·25	0·27
	0 13 58	11·29	11·04	0·25
E. E. of α Ursæ Minoris taken at Pola on the 6th April 1846 Afternoon	0 25 52	38·73	37·89	0·84
	0 23 0	30·64	29·95	0·69
	0 16 30	15·78	15·41	0·37
	0 10 57	6·96	6·77	0·19
	0 3 45	0·82	0·80	0·02
	0 1 56	0·22	0·21	0·01
	0 4 59	1·44	1·40	0·04
	0 8 33	4·24	4·14	0·10
E. E. of α Ursæ Minoris taken at Pola on the 6th April 1846 Afternoon	0 15 26	13·84	13·47	0·37
	0 10 58	6·99	6·81	0·18
	0 3 22	0·66	0·64	0·02
	0 0 21	0·01	0·01	0·00
	0 6 14	2·26	2·20	0·06
	0 7 51	3·56	3·49	0·07
	0 14 43	12·54	12·27	0·27
	0 16 59	16·69	16·32	0·37

B

Traverse Table computed with Spheroidal Azimuths

Stations		Spheroidal Azimuths	Log distance in feet	Co ordinates in feet	
A	B			Direct	Perpendicular
Suktipura	Pagaro	258 · 28 · 15	5.1058512	+ 25503.03	+ 125026.60
Pagaro	Dholpur	176 · 17 · 37	5.1444175	+ 139158.00	− 901.53
Dholpur	Usira	149 · 44 · 13	5.0991994	+ 108535.73	− 63329.31
Usira	Dimdima	82 36 35	5.0953328	− 16020.13	− 123512.26
Dimdima	Machi	27 · 10 24	5.1551817	− 127171.73	− 65282.57
Machi	Nindhar	335 · 42 · 18	5.0124682	− 93798.50	+ 42341.79
Nindhar	Suktiarpura	290 · 32 · 2	6.0065509	− 35609.23	+ 95069.76
				+ 697.17	+ 1298.46

A

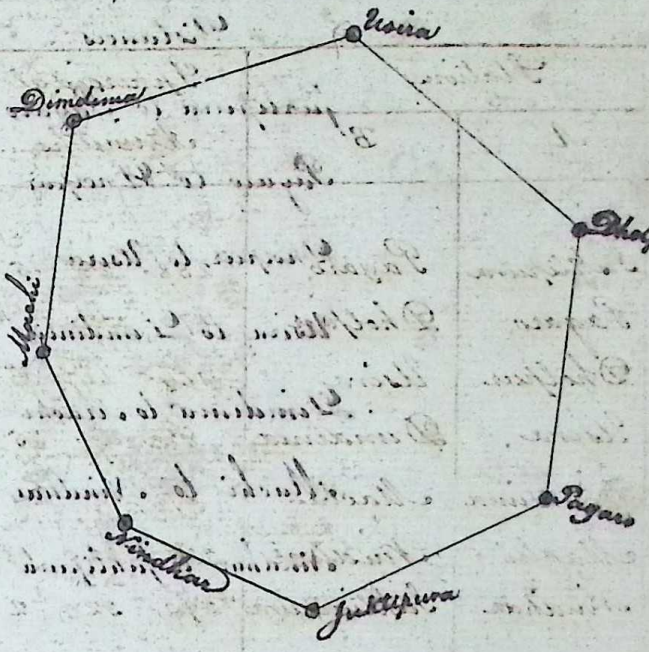
Traverse Table computed with Azimuths deduced from the assumption that the Meridians are parallel straightlines

Stations		Plane Azimuths	Log distance in feet	Co ordinates in feet	
A	B			Direct	Perpendicular
Suktipura	Pagaro	258 · 28 · 15	5.1058512	+ 25503.03	+ 125025.60
Pagaro	Dholpur	176 · 7 28	5.1444175	+ 139130.77	− 9425.35
Dholpur	Usira	149 · 34 · 46	5.0991994	+ 108361.22	− 6336.43
Usira	Dimdima	82 · 32 · 21	5.0953328	− 16172.21	− 123492.43
Dimdima	Machi	27 · 16 24	5.1551817	− 127057.61	− 65504.44
Machi	Nindhar	335 · 53 38	5.0124682	− 93937.59	+ 42032.33
Nindhar	Suktiarpur	290 39 · 54	5.0065509	− 35826.69	+ 94988.04
				+ 0.92	− 3.65

Computed & Compared by
Sreenath Sirdar
& Bholanath Mojoomdar

On the Traverse Table Computation.—

The annexed diagram represents the Girjapahar Heptagon of the Great Arc Series Section Kalianpur – Kalyana.–



The stations forming the Heptagon may be taken as the Revenue Survey Stations forming an entire Circuit, the Survey commencing from Juktipura, and proceeding by Pagaro and Dholpur to close at the origin.–

In the Revenue Survey the elements derived from measurement are of two kinds:– 1st distances and 2nd angles,– which elements with reference to the given heptagon may be taken out from the G. T. Survey Records and treated as measured data.– These data are subjoined below.–

At Juktipura.	∠ Nindhar	& Pagaro	147° 48' 29"
" Pagaro	∠ Juktipura	& Dholpur	97° 39' 13"
" Dholpur	∠ Pagaro	& Usira	163° 27' 18"
" Usira	∠ Dholpur	& Dimdima	112° 57' 35"
" Dimdima	∠ Usira	& Machi	124° 44' 3"
" Machi	∠ Dimdima	& Nindhar	128° 39' 14"
" Nindhar	∠ Machi	& Juktipura	134° 42' 16"

Distances	feet	Log
Juktipura to Pagaro	127600	5·1058507
Pagaro to Dholpur	139449	5·1444169
Dholpur to Usira	125660	5·0991988
Usira to Dimdima	124547	5·0953322
Dimdima to Machi	142949	5·1551811
Machi to Nindhar	102912	5·0124676
Nindhar to Juktipura	101520	5·0065503

Considering these data as the measured elements of the Revenue Survey and also taking the azimuth of Pagaro from Juktipura to be 258° - 28' - 15", the Traverse Table may be computed in the usual way which computation being performed the results will be as given in Table A

A Traverse Table Computed with Bearings deduced from the assumption that the Meridians are parallel Straight Lines.

Stations		Bearings	Log: Distances in feet	Co-ordinates in feet:	
A	B			Direct	Perpendicular
Juktipura	Pagaro	258° 28' 15"	5·1058512	+ 25503·03	+ 126025·60
Pagaro	Dholpur	176° 7' 28"	5·1444176	+ 13930·77	− 9425·35
Dholpur	Usira	149° 34' 46"	5·0991994	+ 10836·22	− 63627·43
Usira	Dimdima	82° 32' 21"	5·0953328	− 16172·21	− 123492·43
Dimdima	Machi	27° 16' 24"	5·1551817	− 127057·61	− 65504·44
Machi	Nindhar	335° 53' 38"	5·0124682	− 93937·59	+ 42032·33
Nindhar	Juktipura	290° 39' 54"	5·0065509	− 35826·69	+ 94988·04
				+ 0·92	− 3·68

On inspection of this Table it will be seen that the direct co-ordinates summed up exhibit an error of 0·92 of a foot and that the perpendicular co-ordinates likewise when added show an error of 3·68 feet. These errors are very minute and may be considered as evanescent.

Now if at the several stations of the Gurjapahar Heptagon, azimuths are observed, they will not agree with the Bearings given in Table A. The observed azimuth in every case will be identical with what in the Trigonometrical Survey is styled the Spheroidal Azimuth. Taking out the spheroidal azimuths at the several stations from the G. T. S. records and regarding them as observed elements, let the Traverse Table be computed afresh. The results of this computation are shewn in Table B on reference to which it will be seen that the direct and perpendicular co-ordinates when added show large errors the former of 597·17 feet and the latter of 1298·46 feet

B

Traverse Table Computed with Spheroidal Azimuths

Stations		Spheroidal Azimuths	Log: Distance in feet	Co-ordinates in feet	
A	B			Direct	Perpendicular
Jukhtipura	Pagaro	258° 28′ 15″	5·1058512	+ 25503·63	+ 126826·60
Pagaro	Dholpur	176° 17′ 37″	5·1444175	+ 139168·00	− 9014·63
Dholpur	Usira	149° 44′ 13″	5·0991994	+ 108536·73	− 63329·31
Usira	Dimdima	82° 36′ 35″	5·0953328	− 16020·13	− 23662·26
Dimdima	Machi	27° 10′ 24″	5·1551817	− 127171·73	− 68252·59
Machi	Nindhar	335° 42′ 18″	5·0124682	− 93798·50	+ 42341·79
Nindhar	Jukhtipura	290° 32′ 2″	5·0065509	− 35609·23	+ 9069·76
				+ 597·17	+ 1298·46

The inference to be drawn from these facts is that only one observed azimuth can be used in the Revenue Survey Computations and that when more than one Observed azimuth are employed the Traverse Table will be discordant.—

Altho this conclusion is made to rest upon facts and actual computation, it may also be easily arrived at by a consideration of the first principles.— The Traverse Table is computed upon the assumption that the meridians are parallel.— This is not a true assumption and the consequence is that every time an azimuth is observed that assumption is militated against, the observed azimuth being referable to the true converging meridians, which observed azimuth therefore when employed in the computation will necessarily destroy the accordancy of the Traverse Table.—

From what is stated above it is clear that when azimuths are observed at two Stations A & B of the Revenue Survey, the former being made the basis of the Traverse Table computation, the latter as it stands, derived from observation cannot be employed in the same deductive process.— That it may become available for such use however it will require to be altered in this way.— Suppose a line to be drawn through Station B parallel to the meridian at A.— It is clear that the Bearing required at B is that which originates from the parallel in question and has nothing to do with the meridian of that station.— Compute therefore the angle between the meridian at B and the parallel drawn through

it and apply it to the observed azimuth, to the end that it may become referable to the parallel in question — the azimuth so altered is the element required in the Traverse Table Computation —

The angle adverted to above may be called the convergancy angle, as it shews the deflection of the true converging meridian from the parallel meridian, — and it can easily be computed as to magnitude and — direction, when the distance from A to B, and the azimuth which one point bears from the other are known. —

The following Table is computed as preparatory to the Calculation of the Graticule Table required for the projection of Mr Nicolson's Hoogli River Triangulation below Calcutta. Scale 1 mile to 1 Inch.–

λ	Radius of Curvature	Normal to the Axis Minor	Radius of the parallel of Lat passing through λ	λ	Meridional Degrees in Miles	Longitudinal Degrees in Miles
20 0	7.318 2362	7.320 7912	7.2937770	20° – 0′	6.8783	6.5016
6	2410	7929	7.2935020	6	6.8784	6.4975
12	2459	7945	7.2932255	12	6.8784	6.4934
18	2508	7961	7.2929475	18	6.8785	6.4892
24	2558	7977	7.2926680	24	6.8786	6.4850
30	2607	7993	7.2923869	30	6.8787	6.4808
36	2657	8011	7.2921046	36	6.8787	6.4766
42	2707	8027	7.2918205	42	6.8788	6.4724
48	2757	8044	7.2915350	48	6.8789	6.4681
54	2806	8060	7.2912479	54	6.8790	6.4639
21 – 0	7.318 2857	7.320 8077	7.2909594	21 – 0	6.8791	6.4596
6	2908	8094	7.2906694	6	6.8791	6.4552
12	2959	8111	7.2903779	12	6.8792	6.4509
18	3010	8128	7.2900848	18	6.8793	6.4466
24	3061	8145	7.2897902	24	6.8794	6.4422
30	3112	8162	7.2894941	30	6.8795	6.4378
36	3164	8180	7.2891966	36	6.8795	6.4334
42	3216	8197	7.2888974	42	6.8796	6.4290
48	3268	8214	7.2885967	48	6.8797	6.4246
54	3320	8232	7.2882946	54	6.8798	6.4200.
22. 0	7.318 3372	7.320 8249	7.2879908	22 – 0	6.8799	6.4156
6	3425	8267	7.2876855	6	6.8800	6.4110
12	3478	8284	7.2873787	12	6.8800	6.4065
18	3531	8302	7.2870704	18	6.8801	6.4020
24	3584	8320	7.2867605	24	6.8802	6.3974
30	3637	8337	7.2864490	30	6.8803	6.3928
36	3690	8355	7.2861361	36	6.8804	6.3882
42	3744	8373	7.2858216	42	6.8805	6.3836
48	3798	8391	7.2855056	48	6.8806	6.3789
54	3852	8409	7.2851879	54	6.8806	6.3743
23 · 0	7.318 3907	7.3208427	7.2848688	23 · 0	6.8807	6.3696

Computed and Compared by
Sreenath Sircar & Bholanath Mojoomdar
12th May 1855

Sides of the squares of 6' of Latitude and Longitude with the Diagonals thereof on the scale of 1 Mile to an Inch

Graticule Table required for the projection of Mr. Nicolson's Hoogli River Triangulation below Calcutta.

Latitude					Values in Inches of			
					m	n	p	q
20	0	to	20	6	6.878	6.502	6.498	9.463
20	6	to	20	12	6.878	6.498	6.493	9.460
20	12	to	20	18	6.878	6.493	6.489	9.457
20	18	to	20	24	6.879	6.489	6.485	9.455
20	24	to	20	30	6.879	6.485	6.481	9.453
20	30	to	20	36	6.879	6.481	6.477	9.450
20	36	to	20	42	6.879	6.477	6.472	9.447
20	42	to	20	48	6.879	6.472	6.468	9.444
20	48	to	20	54	6.879	6.468	6.464	9.441
20	54	to	21	0	6.879	6.464	6.460	9.438
21	0	to	21	6	6.879	6.460	6.455	9.435
21	6	to	21	12	6.879	6.456	6.451	9.432
21	12	to	21	18	6.879	6.451	6.447	9.429
21	18	to	21	24	6.879	6.447	6.442	9.426
21	24	to	21	30	6.879	6.442	6.438	9.423
21	30	to	21	36	6.880	6.438	6.433	9.421
21	36	to	21	42	6.880	6.433	6.429	9.418
21	42	to	21	48	6.880	6.429	6.425	9.415
21	48	to	21	54	6.880	6.425	6.420	9.412
21	54	to	22	0	6.880	6.420	6.416	9.409
22	0	to	22	6	6.880	6.416	6.411	9.406
22	6	to	22	12	6.880	6.411	6.407	9.403
22	12	to	22	18	6.880	6.407	6.402	9.400
22	18	to	22	24	6.880	6.402	6.397	9.396
22	24	to	22	30	6.880	6.397	6.393	9.393
22	30	to	22	36	6.880	6.393	6.388	9.390
22	36	to	22	42	6.880	6.388	6.384	9.387
22	42	to	22	48	6.881	6.384	6.379	9.385
22	48	to	22	54	6.881	6.379	6.374	9.381
22	54	to	23	0	6.881	6.374	6.370	9.378

Computed and Compared by
[illegible] & Bholanath Mujumdar
12 May 1855

Sides of Squares of 4° of Latitude and Longitude with diagonals thereof. -

Latitudes			Values in Miles of			
			m	n	p	q
4°	to	8°	274·8418	275·9776	273·9724	388·7780
8	to	12	274·8945	273·9724	270·6404	386·9302
12	to	16	274·9720	270·6404	265·9972	384·1864
16	to	20	275·0732	265·9972	260·0640	380·5805
20	to	24	275·1960	260·0640	252·8684	376·1580
24	to	28	275·3381	252·8684	244·4436	370·9760
28	to	32	275·4969	244·4436	234·8288	365·1040
32	to	36	275·6690	234·8288	224·0692	358·6243
36	to	40	275·8515	224·0692	212·2152	351·6319

Sides of Squares of 4° of Latitude and Longitude with diagonals thereof on the Scale of 64 Miles to 1 Inch.

Latitude			Values in Inches of			
			m	n	p	q
4°	to	8°	4·294	4·312	4·281	6·075
8	to	12	4·295	4·281	4·229	6·046
12	to	16	4·296	4·229	4·156	6·003
16	to	20	4·298	4·156	4·064	5·947
20	to	24	4·300	4·064	3·951	5·877
24	to	28	4·302	3·951	3·819	5·797
28	to	32	4·305	3·819	3·669	5·705
32	to	36	4·307	3·669	3·501	5·604
36	to	40	4·310	3·501	3·316	5·494

Computed & Compared by
Sreenath Sirdar
& Panchkori Gupta
21st July /59

Arc	Log Sines	Arc	Log Sines	Arc	Log Sines	Arc	Log Sines
0 18 22	7. 660 3043, 416	30 10 12	9. 701 1942, 658	52 57 58	9. 902 1549, 294	79 2 60	9. 992 0200, 737
0 58 53	8. 238 6972, 682	30 25 6	9. 704 4182, 673	53 46 9	9. 906 6810, 275	79 25 7	9. 992 5513, 306
2 42 17	8. 673 8387, 252	31 5 45	9. 713 0459, 895	54 0 29	9. 908 0020, 006	79 56 45	9. 993 2788, 631
2 56 5	8. 709 2544, 529	34 40 0	9. 754 9604, 452	55 35 60	9. 916 5136, 919	80 0 2	9. 993 3522, 014
4 16 8	8. 870 0946, 714	35 32 26	9. 764 3846, 811	56 21 1	9. 920 3532, 978	80 1 26	9. 993 3833, 484
4 16 20	8. 872 1286, 929	35 59 41	9. 769 1636, 180	56 21 2	9. 920 3536, 993	80 10 58	9. 993 5934, 702
5 6 29	8. 949 5575, 209	37 6 28	9. 780 5460, 247	58 43 39	9. 922 2430, 440	80 32 59	9. 994 0655, 943
6 43 4	9. 068 1075, 078	37 17 26	9. 782 3703, 226	61 52 2	9. 945 3982, 971	81 19 43	9. 995 0071, 180
7 10 6	9. 096 1619, 828	37 19 41	9. 782 7433, 272	62 44 45	9. 948 8938, 671	84 59 19	9. 998 3266, 648
7 35 18	9. 120 7533, 846	37 21 12	9. 782 9944, 656	62 47 5	9. 949 0485, 857	85 18 60	9. 998 5476, 426
7 52 49	9. 137 0488, 171	37 31 50	9. 784 7487, 974	64 55 51	9. 957 0308, 529	87 15 36	9. 999 5032, 066
10 33 8	9. 262 7633, 778	40 15 22	9. 810 3705, 706	65 35 38	9. 959 3465, 293	87 15 38	9. 999 5034, 081
12 58 12	9. 351 1018, 943	40 21 11	9. 811 2369, 077	65 53 49	9. 960 3815, 310	87 16 58	9. 999 5114, 351
14 2 6	9. 384 7378, 361	42 33 23	9. 830 1493, 331	67 45 60	9. 966 4470, 913	87 17 32	9. 999 5148, 267
14 37 4	9. 402 0370, 525	42 35 21	9. 830 4197, 795	74 6 37	9. 983 0444, 730	87 48 43	9. 999 6832, 394
18 2 58	9. 491 1341, 351	46 27 60	9. 860 3222, 996	74 30 38	9. 983 9332, 805	89 45 37	9. 999 9961, 986
24 17 63	9. 614 3523, 493	46 32 0	9. 860 8018, 349	74 50 62	9. 984 6329, 682	89 52 25	9. 999 9989, 433
24 18 36	9. 614 5528, 296	47 22 27	9. 880 2290, 793	75 11 32	9. 985 3315, 671	89 52 26	9. 999 9989, 480
26 4 40	9. 643 0486, 303	47 40 41	9. 882 1945, 944	76 2 16	9. 986 9754, 134	89 54 4	9. 999 9993, 531
27 50 2	9. 669 2329, 661	50 34 38	9. 887 8879, 059	76 4 42	9. 987 0517, 275	89 58 20	9. 999 9999, 489
28 30 29	9. 678 7753, 412	51 26 60	9. 893 1419, 365	76 35 23	9. 987 9942, 262	89 58 21	9. 999 9999, 499

26 4 40	9.643 0486, 303	49 - 40 41	9. 882 1945, 944	76 2 - 26	9.986 9754, 134	89 - 54 . 4	9.999 9993, 531
27 - 50 2	9. 669 2329, 561	50 34 38	9. 887 8879, 059	76 - 4 42	9.987 0517, 275	89 - 58 20	9.999 9999, 489
28 30 29	9. 678 7753, 412	51 - 26 60	9. 893 1419, 365	76 - 35 23	9. 987 9942, 262	89 - 58 . 21	9.999 9999, 499

March 1858

Day of the Month	A	Differences 1st	Differences 2nd	B	Differences 1st	Differences 2nd	C	Differences 1st	Differences 2nd	D	Differences 1st	Differences 2nd
1	[illegible]	[illegible]	[illegible]	[illegible]	-3??	+4	[illegible]	[illegible]	-6	[illegible]	[illegible]	[illegible]
2	-1.2517	-24	+14	[illegible]	-282	-14	[illegible]	+55	+1	[illegible]	-64	+3
3	-1.2541	-21	+3	[illegible]	-246	-1.7	[illegible]	+55	0	[illegible]	-67	+2
4	-1.2562	-27	0	+0.7402	-263	-18	-9.1970	+56	-1	-0.1431	-59	+3
5	-1.2598	-19	+2	+0.7122	-281	-20	-9.1915	+56	+2	-0.1487	-56	+2
6	-1.2602			+0.6821	-301	18	-9.1868	+57		-0.1541	-64	

April 1858

Day of the Month	A	Differences 1st	Differences 2nd	B	Differences 1st	Differences 2nd	C	Differences 1st	Differences 2nd	D	Differences 1st	Differences 2nd
1	-1.2627	[illegible]	[illegible]	[illegible]	[illegible]	+3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	-1.2610	+17	+1	[illegible]	-332	+25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	-1.2592	+18	+1	[illegible]	-307	+21	[illegible]	[illegible]	0	[illegible]	[illegible]	+2
4	-1.2578	+19	+2	-0.7267	-286	+19	[illegible]	+89	+4	-0.2196	+2	+2
5	-1.2552	+21	+1	-0.7534	-267	+17	-8.9766	+93	+4	-0.2191	+4	[illegible]
6	-1.2530	+22		-0.7784	-250		-8.9669	+97		-0.2187	+4	

May 1853

Day of the Month	A	Differences 1st	Differences 2nd	B	Differences 1st	Differences 2nd	C	Differences 1st	Differences 2nd	D	Differences 1st	Differences 2nd
1	−1.1472			−1.1312			−8.6[illegible]			−0.1707		
2	−1.1406	+66	+2	−1.1298	−81	+3	−8.4819	+363	+37	−0.1678	+29	+1
3	−1.1338	+68	+3	−1.1472	−79	+3	−8.4519	+400	+43	−0.1642	+30	−1
4	−1.1267	+71	+2	−1.1548	−76	+2	−8.4076	+443	+59	−0.1619	+29	+1
5	−1.1194	+73	+3	−1.1622	−74	+3	−8.3574	+502	+73	−0.1589	+30	+1
6	−1.1118	+76		−1.1693	−71		−8.2999	+575		−0.1558	+31	

June 18

Day of the Month	A	Differences 1st	Differences 2nd	B	Differences 1st	Differences 2nd	C	Differences 1st	Differences 2nd	D	Differences 1st	Differences 2nd
1	−0.7781			−1.2866			+8.7962			−0.0940		
2	−0.7560	+221	+13	−1.2890	−24	+2	+8.8196	+234	−11	−0.0932	+8	−1
3	−0.7326	+234	+14	−1.2912	−22	+1	+8.8419	+223	−9	−0.0925	+7	−2
4	−0.7078	+248	+17	−1.2933	−21	+1	+8.8633	+214	−10	−0.0920	+5	−2
5	−0.6813	+265	+18	−1.2953	−20	+1	+8.8837	+204	−7	−0.0917	+3	−2
6	−0.6530	+283		−1.2972	−19		+8.9034	+197		−0.0916	+1	

4	−0·7078	+248	+17	−1·2933	−21	+1	+8·8633	+21·4	−10	−0·0920	+5	−2
5	−0·6813	+265	+18	−1·2953	−20	+1	+8·8837	+20·4	−7	−0·0917	+3	−2
6	−0·6530	+283		−1·2973	−19		8·9034	+197		−0·0916	+1	

July 1853

Day of the Month	A	Differences 1st	Differences 2nd	B	Differences 1st	Differences 2nd	C	Differences 1st	Differences 2nd	D	Differences 1st	Differences 2nd
1	+0·5108			−1·3035			+9·2312			−0·1501		
2	+0·5493	+392	−33	−1·3021	+14	−0	+9·2403	+91	−3	−0·1546	−45	+2
3	+0·5857	+359	−28	−1·3007	+14	+2	+9·2491	+88	−1	−0·1593	−47	0
4	+0·6188	+331	−25	−1·2991	+16	+1	+9·2578	+87	−3	−0·1640	−47	+2
5	+0·6494	+306	−22	−1·2974	+17	+1	+9·2662	+84	−2	−0·1689	−49	0
6	+0·6778	+284		−1·2956	+18		+9·2744	+82		−0·1738	−49	

August 1853

Day of the Month	A	Differences 1st	Differences 2nd	B	Differences 1st	Differences 2nd	C	Differences 1st	Differences 2nd	D	Differences 1st	Differences 2nd
1	+1·0766			−1·1971			+9·4369			−0·3178		
2	+1·0852	+86	−2	−1·1910	+61	+3	+9·4416	+47	−1	−0·3203	−55	+2
3	+1·0936	+84	−3	−1·1846	+64	0	+9·4462	+46	−1	−0·3286	−53	−1
4	+1·1017	+81	−3	−1·1782	+64	+4	+9·4507	+45	−1	−0·3340	−54	+1
5	+1·1095	+78	−2	−1·1714	+68	+1	+9·4551	+44	−1	−0·3393	−53	+1
6	+1·1171	+76		−1·1645	+69		+9·4594	+43		−0·3445	−52	

September 1853

Day of the Month	A	Differences 1st	Differences 2nd	B	Differences 1st	Differences 2nd	C	Differences 1st	Differences 2nd	D	Differences 1st	Differences 2nd
1	[illegible]			[illegible]			[illegible]			[illegible]		
		[illegible]			[illegible]			+27			[illegible]	
2	[illegible]		[illegible]	[illegible]		+18	[illegible]		+1	[illegible]		[illegible]
		[illegible]			+262			+26			−30	
3	+1.2491		−1	−0.8152		+12	+9.5529		0	−0.4603		+1
		+25			+224			+26			−29	
4	+1.2516		−3	−0.7928		+14	+9.5555		0	−0.4632		+2
		+22			+238			+26			−27	
5	+1.2538		0	−0.7690		+15	+9.5581		−1	−0.4659		0
		+22			+253			+25			−27	
6	+1.2660			−0.7437			+9.5606			−0.4686		

October 1853

Day of the Month	A	Differences 1st	Differences 2nd	B	Differences 1st	Differences 2nd	C	Differences 1st	Differences 2nd	D	Differences 1st	Differences 2nd
1	+1.2876			+0.4899			+9.6169			−0.5053		
		−12			+462			+21			−3	
2	+1.2864		−2	+0.5361		−45	+9.6190		0	−0.5056		+1
		−14			+417			+21			−2	
3	+1.2850		−1	+5.5778		−38	+9.6211		0	−0.5058		+1
		−15			+379			+21			−1	
4	+1.2635		−1	+0.6157		−31	+9.6232		0	−0.5059		0
		−16			+348			+21			−1	
5	+1.2619		−1	+0.6505		−28	+9.6253		0	−0.5060		0
		−17			+320			+21			+1	
6	+1.2602			+0.6825			+9.6274			−0.5059		

Memorandum of Instructions for inspecting a Party of the Gt. Trigl. Survey.

Instruments

Examine and report on the following points.

Great Theodolite. – 1st. Its general condition and appearance as to cleanliness and freedom from defect and blemish.

2nd. The Stand. – Its freedom from shake and tightness of the screws. – Unless the stand be immoveable, the observations will be bad.

3d. The feet Screws. – Lift each foot screw successively by gently tilting the theodolite so as to make it rest on the 2 other feet. Then shake the free screw and if it be loose, tighten it gently, so that it may be firm yet capable of being moved smoothly. – If the feet screws work loose, the observations will be bad.

4th. The Axis of Motion. – If the instrument does not work glibly, the stiffness may be occasioned by want of cleaning and lubrication; – Clean and lubricate accordingly, and if the axis be still stiff, investigate the reason. In the case of instruments having Ramsden's Axis, in which friction is relieved by an Adjustable pivot or socket, it is always a matter of difficulty to adjust the Axis so as to avoid stiffness on the one hand and wabbling on the other, the latter evil being shewn by difficulty in levelling and wild differences in reading the microscopes as the instrument is turned from left to right. The same position does not answer generally for all temperatures, but on the Coast seasonal changes of temperature cannot be great. The excellence of the observations depends on the Axis being set judiciously so as to avoid extremes.

6th. The tangent screws should work glibly and at the same time be free from shake, and not liable to press on

the

the limb so as to put the level out.

6th. The Horizontal Microscopes.

See that the glasses are perfectly clean, the vision clear and distinct, and free from Parallax and the Micrometers clean and working glib and steady, without jirk on releasing. Examine that the wires be straight clean firm, and not liable to catch while moving over the Central 10 minutes. See also that the Micrometer is set tangentically to the circle. Cause the runs to be taken at a medium temperature and correct one or more Microscopes if necessary. The head of the party should take the runs, while the inspecting Officer examines his proceedings.

7th. See that the Transit Axis fits exactly between the Ys without being too tight on the one hand, or too loose on the other. The face of each Y must be parallel to the shoulders of the Axis. This can be judged of by the grinding marks left by the shoulders on the Y faces. If the Ys fit so tight as to act on the shoulders like a spring the telescope will not describe true Verticals, for one or other pivot will be liable to rise off its seat, and thus the horizontal Ls will be vitiated. As far as these are concerned perfect freedom or looseness between the shoulders and Ys is preferable, but if there be too much play, the runs of the Vertical Microscopes will be vitiated. Therefore to suit the requirements of both kinds of observations the Axis should be quite free, so that the pivots may rest firmly on their seats, and the degree of play between the shoulders and Y faces should be as small as practicable, but some freedom must exist. Its effect on the Vertical Microscope runs, should however be almost insensible and may be eliminated by placing one shoulder in contact with one Y and taking care the vertical clamp does not affect this condition. If an Instrument has been used some time with the Ys too tight on the Axis shoulders, the latter will have bored into one or both faces. In this case, the axis will sit on the Angle of the shoulders, instead of on the pivots, if there be any indications of this sort, the Y face must be filed smooth with

a fine file, taking care to remove a little more of the surface of the Y face below the point A of the shoulder, so that there may be no resistance to the pivot sitting down. It is to be observed, if 2 surfaces of equal size grind on each other, they will work smooth, but if a small surface works against a large one, the former will always bore into the latter.

8th Examine whether the Object & Eye Glasses of the telescope are clean and the Cobweb lines in the focus straight, clean and tight, i.e. free from festoons. If not, fresh lines should be fixed.

9th See that the Solar Focus of the telescope has been correctly determined and marked. With this view about 4 or 5 hours P.M. direct the telescope on a well-defined object, such as a heliotrope, and observe if there be any parallax. If the image moves with the eye, it is situated between the wires and object glass; and the telescope must be shortened. If the image moves contrary to the motion of the eye, it is between the wires and the eye, or the telescope is too short. The Solar focus may also be tested on the Pole Star.

10th Having levelled the Instrument and transit axis, see whether the Vertical wire is truly Vertical. For this purpose at 4 or 5 hours P.M. intersect a steady mark, the Referring Heliotrope for example, and move the telescope up and down, so that the Heliotrope may run along the wire. If the object remains intersected at all parts of the wire, the latter is vertical, otherwise the wire plate must be turned round till verticality is insured.

11th The horizontality of the other wire may be tested on similar principles by intersecting an object on the horizontal wire, and then moving the Instrument in Azimuth, so that the object may run along that wire. If the intersection remains perfect, the wire is horizontal otherwise it is not at right angles to the Vertical wire, a defect to be remedied on the application of new wires.

12th Having set the wires true it is to be explained that the intersections are nevertheless always to be made on one constant part of the wire, as a precaution against the adjustment getting out and also to ensure the habit of observing this way.

any rate
tical one

13th. Next try the Horizontal Collimation. At 4 or 5 hours P. M. intersect an object, and clamping the horizontal Circle read the Microscopes. Then lift the telescope out of the Vs and replace it with reversed pivots. If the object be intersected in this second position, there is no Collimation Error, otherwise reintersect and read the Microscopes again. Half the difference is the measure of the Error which record accordingly. To correct the Error practically, place the Instrument to the mean reading and intersect the object by moving the wires by means of the screws that act on the wire plates. These are generally drawing screws and the motion they impart is the same as the apparent one required. For example if the wire has to be moved apparently to the right release the left screw and tighten the right. As the Instrument is liable to be shaken and its position deranged while the telescope is being reversed on its pivots, the adjustment must be repeated several times, until there is every reason to be confident of the result.

14th. Examine the Vertical Microscopes as described under para 6 for clearness of glasses, steadiness and perfection of motion in the Micrometer, its tangentical position and the firmness of the wires. Also ascertain the runs to be taken in the horizontal position of the telescope face right and face left.

15. In taking Verticals, if the same division is intersected on both faces, the Vertical angle will be free from graduation error. This is the case when it does not exceed say 3'. If the Vertical ∠ exceeds 3' and falls short of 6' take the horizontal stroke on the first face and the nearest stroke on the other. Then repeat the observation taking the horizontal stroke on the 2nd face and the nearest stroke on the other. By this means the result will depend on a mean of 2 sets of Divisions and the error of runs will in a great

measure be eliminated. As a general rule for other values of Vertical L.s, the nearest stroke to the Zero of the Microscope should first be taken, and on the other face, the nearest stroke to the wires, so that the Micrometer may have to be moved as little as possible. If the stroke intersected be about 2 to 3 Minutes from Zero, when the observation is repeated, move the Micrometer to the division on the other side of Zero, and on reversing to the first face, take the nearest division. Thus you will employ twice the ordinary number of Divisions with the smallest movement of Micrometer.

16th. Try the runs of the 2 levels with the usual precautions as laid down in Department rules.

17th. Examine the Graduation of the Circles and see that they are uninjured, or record any apparent defects.

Small or Vernier Theodolites

18th. The examination of these is precisely similar to the foregoing, only there are no Micrometers.

19th. Examine the Verniers and Graduation.

20th. In addition to the Axis of Horizontal motion being clean and properly lubricated, the socket of the Vertical Verniers, which works on the transit Axis, must likewise be cleaned and lubricated.

21st. The condition and rate of Chronometer to be noted.

22nd. Heliotropes, Lamps, Perambulators, Sight Vanes, and other apparatus to be examined and reported on.

Observations

23d. An examination of the Angle Books will give a fair Criterion of the quality of the observations, but in addition to this test, the inspecting Officer will examine the observations in progress and judge of their style and accuracy. It will be an advantage if the inspection be made at the time Pole Stars are being observed.

24th. Note whether the Stations either towers or platforms, are properly built and isolated, according to departmental rules. Unless the instrument stands on a firm base, good observations cannot be expected. It is also to be noted whether the observer is in the habit of centering carefully, protecting the Instrument against the Sun, wind and all influences which would deteriorate the observations. At night the Angle book keepers lamp should not be near a ray, because of the agitation produced by smoke. The same remarks apply to the light kept outside for igniting tapers. It is further to be noticed whether the Observatory is strong, well pitched and every precaution taken against accidents. The head of the party should always superintend personally the packing and unpacking of the Instrument.

Public Property

25th. To be examined with the last Official Returns.

Professional Records

26th. The Angle Books should be clean and

neatly written with good ink free from scratching and blots, and brought up in duplicate. They must be signed on each page as examined and Compared. The descriptions of Stations entered and signed, with the keys of the Micrometers and such other notes and information as may be requisite to enable the work to be brought up. On these points, as well as the state and progress of the Computations, due report to be made.

Office Books & Accounts

27th The state of Pay to be examined and reported on. The Acquittance Rolls should be carefully entered with Payments complete to the last issue of Pay, and signed as correct by the Officers in charge.

Native Establishment

28th The Officer in Charge of a party to furnish the inspecting Officer with a present state, noting those present and those absent on duty, the nature of which is to be specified. – The Establishment will be mustered and compared with the present state roll; enquiry is then to be made whether the Pay has been disbursed up to the last issue of Pay, specifying the Month. – Also whether any one wishes to complain. In the event of Complaints being preferred, they are to be enquired into and disposed of, either by summary decision on the spot, or reference to Head Quarters. It is to be noted by the inspecting Officer, on the present state whether the men are smart, able bodied, intelligent, and of a description suitable to the Pay. The qualifications

of

of the Tindal, Jemadar, Naik, Daffadars, and Heliotropers to be carefully remarked

Civil Establishment

29th. Enquiry to be made whether the head of the party and his assistants are mutually pleased and contented with each other. – Any Complaints to be disposed of as circumstances may render necessary.

General Remarks

30th. The general style and Character of the operations and mode of doing business to be reported on; also whether the people of the Country are satisfied

To

J. Peyton Esqr.
Chief Civil Offr. G.T.S.

Sd. A.S. Waugh Lieut. Colonel
Surveyor General of India

Camp Allahabad

Serony Quadrilateral

Statement of the Results of the Reduction of the Serony Base Line Quadrilaterals agreeably to the method of Minimum Squares.

Triangular Equations.

Pardho				
Baovasa	93	7	52	384,79
Kalianpur	34	22	28	138,30
	52	29	40	970,89
	180	0	1	493,98
π + Spherical Excess	180	0	1	494
Kamkhera	87	54	37	335,45
Kalianpur	67	21	28	628,65
Baovasa	24	45	55	322,90
	180	0	1	287,00
π + Spherical Excess	180	0	1	287,5
Kalianpur	119	51	9	599,58
Pardho	25	8	13	488,57
Kamkhera	35	0	37	595,86
	180	0	0	684,01
π + Spherical Excess	180	0	0	684,
Baovasa	89	6	23	461,19
Kamkhera	52	53	59	739,57
Pardho	67	59	38	896,23
	180	0	2	096,99
π + Spherical Excess	180	0	2	097,
Kalianpur	144	16	37	448,20
Surantal	18	59	29	907,72
Kamkhera	16	43	32	948,08
	180	0	0	301,00
π + Spherical Excess	180	0	0	301,

Surony Quadrilateral

Baorasa	46	13	0	315, 14
Kambhera	71	11	4	390, 86
Surantal	62	35	57	481, 51
	180	0	2	187, 00
π + Spherical Excess	180	0	2	187,
Surantal	81	36	27	389, 22
Baorasa	21	29	4	992, 25
Kalianpur	76	55	28	819, 51
	180	0	1	200, 98
π + Spherical Excess	180	0	1	201,
N. End Base	165	37	58	248, 52
Baorasa	7	41	19	996, 68
Kalianpur	6	40	41	960, 80
	180	0	0	209, 00
π + Spherical Excess	180	0	0	209,
Kalianpur	74	2	10	592, 46
N. End Base	48	30	23	979, 44
Kambhera	62	27	26	151, 10
	180	0	0	723, 00
π + Spherical Excess	180	0	0	723,
Baorasa	32	25	15	819, 67
Kambhera	25	27	11	184, 34
N. End Base	122	7	34	269, 08
	180	0	0	772, 99
π + Spherical Excess	180	0	0	773,
N. End Base	154	17	9	158, 26
Baorasa	13	47	44	998, 56
Surantal	11	55	6	213, 19
	180	0	0	367, 01
π + Spherical Excess	180	0	0	367,

Serony Quadrilateral

N. End Base	83	35	16·572,67
Surantal	50	40	51·268,31
Hamkhera	45	43	53·206,03
	180	0	1·047,01
π + Spherical Excess	180	0	1·047,
N. End Base	40	4	52·593,26
Surantal	69	40	21·176,03
Kalianpur	70	14	46·855,72
	180	0	0·625,01
π + Spherical Excess	180	0	0·625,
S. End Base	132	42	27·558,19
Surantal	15	45	20·066,00
Hamkhera	31	32	12·738,81
	180	0	0·363,00
π + Spherical Excess	180	0	0·363,
S. End Base	73	39	15·249,56
N. End Base	71	28	14·100,13
Surantal	34	53	31·202,30
	180	0	0·551,99
π + Spherical Excess	180	0	0·552,
S. End Base	153	35	17·192,24
Hamkhera	14	11	40·467,22
N. End Base	12	10	2·472,53
	180	0	0·131,99
π + Spherical Excess	180	0	0·132,

Serong Quadrilateral

Toto-partial and Central Equations

Pardho Angles

Between	Kalianpur & Kamkhera	25	8	13·488,57
	Kamkhera & Baorasa	67	59	38·896,23
		93	7	52·384,80
	Kalianpur & Baorasa	93	7	52·384,79

Swantal Angles

Between	Baorasa & Kamkhera	62	35	57·481,51
	Kamkhera & Kalianpur	18	59	29·907,72
		81	35	27·389,23
	Baorasa & Kalianpur	81	35	27·389,22
Between	Baorasa & N. End Base	11	55	6·213,19
	N. End Base & Kamkhera	50	40	51·268,31
		62	35	57·481,50
	Baorasa & Kamkhera	62	35	57·481,51
Between	Baorasa & N. End Base	11	55	6·213,19
	N. End Base & Kalianpur	69	40	21·176,03
		81	35	27·389,22
	Baorasa & Kalianpur	81	35	27·389,22
Between	N. End Base & Kamkhera	50	40	51·268,31
	Kamkhera & Kalianpur	18	59	29·907,72
		69	40	21·176,03
	N. End Base & Kalianpur	69	40	21·176,03
Between	N. End Base & S. End Base	34	55	31·202,30
	S. End Base & Kamkhera	15	45	20·066,00
		50	40	51·268,30
	N. End Base & Kamkhera	50	40	51·268,31

Kalianpur Angles

Between	Pardho & Baorasa	52	29	40·970,89
	Baorasa & Kamkhera	67	21	28·628,65
		119	51	9·599,54
	Pardho & Kamkhera	119	51	9·599,53

Serving Quadrilateral

Between Surantal & Baarasa	76	55	28	819,61
Baarasa & Kamkhera	67	21	28	628,65
	144	16	57	448,16
Surantal & Kamkhera	144	16	57	448,20
Between N. End Base & Baarasa	6	40	41	968,80
Baarasa & Kamkhera	67	21	28	628,65
	74	2	10	592,46
N. End Base & Kamkhera	74	2	10	592,46
Between Surantal & N. End Base	70	14	46	868,72
N. End Base & Baarasa	6	40	41	968,80
	76	55	28	819,52
Surantal & Baarasa	76	55	28	819,61
Between Surantal & N. End Base	70	14	46	868,72
N. End Base & Kamkhera	74	2	10	592,46
	144	16	57	448,18
Surantal & Kamkhera	144	16	57	448,20

Baarasa Angles.

Between Pardha & Halianpur	34	22	28	196,30
Halianpur & Kamkhera	24	43	55	322,90
	59	6	23	461,20
Pardha & Kamkhera	59	6	23	461,19
Between Surantal & Halianpur	21	29	41	992,25
Halianpur & Kamkhera	24	43	55	322,90
	46	13	0	315,15
Surantal & Kamkhera	46	13	0	315,14
Between N. End Base & Halianpur	7	41	19	996,68
Halianpur & Kamkhera	24	43	55	322,90
	32	25	15	319,58
N. End Base & Kamkhera	32	25	15	319,57

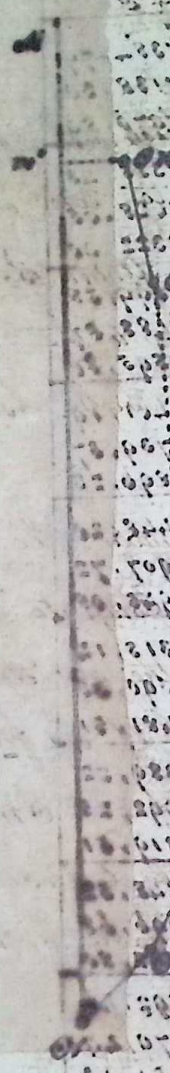
Sironj Quadrilateral

Between Surantal & N. End Base			13 · 47 · 44·995,86	
N. End Base & Kamkhera			32 · 25 · 15·319,37	
			46 · 13 · 0·315,19	
Surantal & Kamkhera			46 · 13 · 0·314,14	
Between Surantal & N. End Base			13 · 47 · 44·995,86	
N. End Base & Kalianpur			7 · 41 · 19·996,68	
			21 · 29 · 4·992,24	
Surantal & Kalianpur			21 · 29 · 4·992,25	
	Kamkhera Angles			
Between Baorasa & Pardho			52 · 53 · 59·739,57	
Pardho & Kalianpur			35 · 0 · 37·595,86	
			87 · 54 · 37·335,43	
Baorasa & Kalianpur			87 · 54 · 37·335,45	
Between Baorasa & Surantal			71 · 11 · 4·390,35	
Surantal & Kalianpur			16 · 43 · 32·946,08	
			87 · 54 · 37·335,43	
Baorasa & Kalianpur			87 · 54 · 37·335,45	
Between Baorasa & N. End Base			25 · 27 · 11·184,04	
N. End Base & Kalianpur			62 · 27 · 26·151,10	
			87 · 54 · 37·335,44	
Baorasa & Kalianpur			87 · 54 · 37·335,45	
Between Baorasa & N. End Base			25 · 27 · 11·184,04	
N. End Base & Surantal			46 · 43 · 53·206,03	
			71 · 11 · 4·390,37	
Baorasa & Surantal			71 · 11 · 4·390,35	
Between N. End Base & Surantal			45 · 43 · 53·206,03	
Surantal & Kalianpur			16 · 43 · 32·945,08	
N. End Base & Kalianpur			62 · 27 · 26·151,11	
			62 · 27 · 26·151,10	

Seronj Quadrilateral
Final Results

Names of Stations	Observed Angles	Compensation of Error	Spheroidal Angles
Pardho	93° 7' 52"391	− ·00621	93° 7' 52"384,79
Baovasa	34 22 28·226	− ·08770	34 22 28·138,30
Kalianpur	52 29 41·031	− ·06011	52 29 40·970,89
Kamkhera	87 54 36·798	+ ·53745	87 54 37·385,45
Kalianpur	67 21 28·511	+ ·11765	67 21 28·628,65
Baovasa	24 43 54·936	+ ·38690	24 43 55·322,90
Kalianpur	119 51 9·542	+ ·05788	119 51 9·599,58
Pardho	25 8 13·880	− ·39143	25 8 13·488,57
Kamkhera	35 0 37·441	+ ·15486	35 0 37·595,86
Baovasa	59 6 23·163	+ ·29819	59 6 23·461,19
Kamkhera	52 53 39·358	+ ·38157	52 53 39·739,57
Pardho	67 59 38·511	+ ·38523	67 59 38·896,23
Kalianpur	144 16 57·639	− ·19080	144 16 57·448,20
Surantal	18 59 30·709	− ·80128	18 59 29·907,72
Kamkhera	16 43 32·793	+ ·15208	16 43 32·945,08
Baovasa	46 13 0·407	− ·09186	46 13 0·315,14
Kamkhera	71 11 4·005	+ ·38535	71 11 4·390,35
Surantal	62 35 57·125	+ ·35651	62 35 57·481,51
Surantal	81 35 27·834	− ·44478	81 35 27·389,22
Baovasa	21 29 5·470	− ·47775	21 29 4·992,25
Kalianpur	76 55 29·128	− ·30849	77 55 28·819,51
N. End Base	165 37 57·165	+ 1·08352	165 37 58·248,52
Baovasa	7 41 19·486	+ ·51068	7 41 19·996,68
Kalianpur	6 40 41·792	+ ·17180	6 40 41·[illegible],80
Kalianpur	74 2 10·303	+ ·28946	74 2 10·592,46
N. End Base	43 30 23·345	+ ·63444	43 30 23·979,44
Kamkhera	62 27 25·599	+ ·55210	62 27 26·151,10
Baovasa	32 25 14·423	+ ·89657	32 25 15·319,59
Kamkhera	25 27 11·199	− ·01468	25 27 11·184,34
N. End Base	122 7 33·820	+ ·44908	122 7 34·269,08
N. End Base	154 17 10·054	− ·89574	154 17 9·158,26
Baovasa	13 47 45·984	− ·98844	13 47 44·995,56
Surantal	11 55 6·396	− ·18281	11 55 6·213,19
N. End Base	83 35 16·126	+ ·44667	83 35 16·572,67
Surantal	50 40 50·729	+ ·53931	50 40 51·268,31
Kamkhera	45 43 52·806	+ ·40003	45 43 53·206,03
N. End Base	40 4 52·781	− ·18774	40 4 52·593,24
Surantal	69 40 21·438	− ·26197	69 40 21·176,03
Kalianpur	70 14 47·336	− ·48028	70 14 46·855,72
S. End Base	132 42 28·030	− ·47181	132 42 27·558,19
Surantal	15 45 20·057	+ ·00900	15 45 20·066,00
Kamkhera	31 32 12·856	− ·11619	31 32 12·738,81
S. End Base	73 39 15·047	+ ·20236	73 39 15·249,56
N. End Base	71 25 12·832	+ ·26803	71 25 14·100,13
Surantal	34 55 30·672	+ ·53030	34 55 31·202,30
S. End Base	153 38 16·932	+ ·26924	153 38 17·192,24
Kamkhera	14 11 39·951	+ ·51682	14 11 40·467,22
N. End Base	12 10 2·894	+ ·17553	12 10 2·472,53

On the Method of Comparing Observed Azimuths taken at different Stations of a Trigonometrical Survey.



Suppose the Survey to be a Meridional Series. In diagram No. 1. O1 & On represent Azimuth Stations, the intermediate stations O2, O3, O4 ..... connecting them being small points of the Triangulation as lie nearest to the meridian O1, M of O1. From On draw the perpendicular On, m to the meridian. Let the points O1, O2, O3 ..... On lie above the Meridian. Now the distances O1 O2, O2 to O3, O3 to O4 ..... On-1 to On are known as likewise the angles at O1, O2, O3 ..... On-1. Considering these points as Route Survey stations, compute the direct & perpendicular co-ordinates corresponding to them the meridian line O1, M being taken as the axis of direct co-ordinates, it is clear that the perpendicular On, m will be the perpendicular co-ordinate of On.

The preceeding computations being performed deduce the area of the figure O1 O2 O3 ..... On, & m which being multiplied by $\frac{\text{Cosec } 1^{\circ}}{V^2}$ will furnish its Spherical excess.

In the foregoing computations no error will be entailed by taking the angles to the nearest second & the co-ordinates to the nearest foot.

On referring to the diagram it will appear that the Route Survey angles O1, O2, O3, ... On are the

interior Angles of the Polygon 01 02 03 ..... On & m' the number of whose sides is (n+1) .—

Now 01 + 02 + 03 ..... + On + m' = (n-1)π + e

But m' = ½π Hence

01 + 02 + 03 ..... + On = $\frac{(2n-3)\pi}{2}$ + e which will give On.

In this deduction the Angles used must be taken to two or three decimals of a second, as may be given in the Angle Book On is the Angle, between O n-1 and the perpendicular to the meridian paseing through 01, deduced from the Azimuth observation at 01.—

Now to compare this term with the result of the Azimuth observations taken at On, Compute the Angle which the perpendicular On, m' makes with the meridian of On, or which is the same thing, taking the Azimuth of On from m' to be 270, deduce the back Azimuth of m' from On which back Azimuth combined with On previously deduced will give the Azimuth of O n-1 which is comparable with the observed value of the Azimuth taken at On.—

Now put $\frac{1}{\frac{1}{3}(r' + r'' + r''')} = R$ which is the proper radius of the triangle, and we get. -

$$E'' = \frac{a.b.\sin C \times \text{cosec } 1''}{2R^2}$$

$$\frac{a.b.\sin C \times \text{cosec } 1''}{2\gamma^2} \times \left\{1 + \frac{2}{3}\cdot\frac{(\gamma - r)}{r}(\cos^2\theta' + \cos^2\theta'' + \cos^2\theta''')\right\}$$

$$= e'' + e'' \times \frac{2}{3}\cdot\frac{(\gamma - r)}{r}\{\cos^2\theta' + \cos^2\theta'' + \cos^2\theta'''\}$$

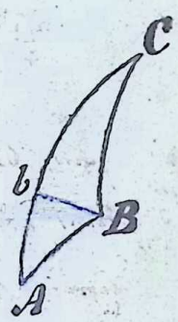
in which $$e'' = \frac{a.b.\sin C \times \text{cosec } 1''}{2\gamma^2}$$

Let A and B be two stations of a Trigonometrical Survey whose latitudes and longitudes are given.- It is required to determine the perpendicular distance of B from the meridian of A and also the part of the same Meridian intercepted between the perpendicular in question and the station A.-

Designate as follows the given elements.-

λ = Latitude of A, ... L Longitude of A,

λ' = d° of B, ... L' d° of B,



In the annexed diagram A & B represent the two given stations and C the earth's pole.- Joining CA & CB by arcs of great circles it will follow.-

$CA = \frac{1}{2}\pi - \lambda$ .. Colatitude of A.

$CB = \frac{1}{2}\pi - \lambda'$ d° of B

$\angle C = L' \sim L$ Difference of longitudes between A & B.

Draw Bb perpendicular to CA which is the meridian of A, Now the arc Bb is very nerely equal to $\angle C$ and hence it may be assumed that $Bb = C + dC$. Also the arc CB is very nearly equivalent to Cb or which is the same thing the latitudes of B & b are very nearly equal wherefore denoting by $\lambda' + d\lambda$ the latitude of b, there will result

$Cb = \frac{1}{2}\pi - \lambda' - d\lambda$ .... Colatitude of b.-

Now to determine dC.- In the right angled triangle CbB, the following equation obtains,

$$\sin Bb = \sin CB \cdot \sin C$$

$$\text{or } \sin(C + dC) = \cos\lambda' \cdot \sin C$$

But $\sin(C + dC) = \sin C + \cos C \cdot dC$

Wherefore $\cos C \cdot dC = (\cos\lambda' - 1) \cdot \sin C$

$$\text{or } dC = -2\sin^2\tfrac{1}{2}\lambda' \cdot \tan C$$

Again to determine $d\lambda$.— In the same right angled triangled CbB take the following equation.—

$$\cos C = \tan Cb \cdot \cot CB$$

$$\text{or } \cos C = \cot(\lambda' + d\lambda) \cdot \tan \lambda'$$

$$\text{or } \tan(\lambda' + d\lambda) = \frac{\tan \lambda'}{\cos C}$$

$$\text{but } \tan(\lambda' + d\lambda) = \tan \lambda' + \sec^2 \lambda' \cdot d\lambda$$

$$\text{wherefore } \sec^2 \lambda' \cdot d\lambda = \frac{(1 - \cos C) \tan \lambda'}{\cos C}$$

$$d\lambda = \sin 2\lambda' \cdot \sec C = \sin^2 \tfrac{1}{2} C$$

In this investigation the development of $\sin(C + dC)$ is made to stop at the first power of dC which may not always give a sufficiently exact result. To do away with this limitation let the developement be extended to the 2nd power of dC which may be done in the following manner.—

$$\sin(C + dC) = \sin C \cdot \cos dC + \cos C \cdot \sin dC =$$

$$\sin C \cdot (1 - \tfrac{1}{2} dC^2) + \cos C \cdot dC$$

which arranged according to the powers of dC will give

$$\sin C + \cos C \cdot dC - \tfrac{1}{2} \sin C \cdot dC^2 = \cos \lambda' \cdot \sin C$$

or

$$dC - \tfrac{1}{2} \tan C \cdot dC^2 = -2 \sin^2 \tfrac{1}{2} \lambda' \cdot \tan C.$$

Here $\alpha = 1$, $\beta = -\tfrac{1}{2} \tan C$,

$$\Phi = -2 \sin^2 \tfrac{1}{2} \lambda' \cdot \tan C, \quad \Psi = 0$$

Wherefore $A = \frac{\Phi}{\alpha} = -2 \sin^2 \tfrac{1}{2} \lambda' \cdot \tan C$

$$B = \frac{\Psi}{\alpha} - \frac{\beta \cdot \Phi^2}{\alpha^3} = 2 \tan^3 C \cdot \sin^4 \tfrac{1}{2} \lambda'$$

Hence $dC = -2 \sin^2 \tfrac{1}{2} \lambda' \cdot \tan C + 2 \tan^3 C \cdot \sin^4 \tfrac{1}{2} \lambda'$

which is perhaps as rigorous a value as will ever be required in practice.—

In likemanner the development of $\tan(\lambda' + d\lambda)$

may be made to include the 2nd power of $d\lambda$ thus:-

$$\tan(\lambda' + d\lambda) = \tan\lambda' + \sec^2\lambda' . d\lambda + \sec^2\lambda' . \tan\lambda' . d\lambda^2$$

Hence

$$\sec^2\lambda' . d\lambda + \tfrac{1}{2}\sec^2\lambda' . \tan\lambda' . d\lambda^2 = \tan\lambda'(\tan C - 1) = 2\tan\lambda' . \sin^2\tfrac{1}{2}C . \sec C$$

$$d\lambda + \tan\lambda' . d\lambda^2 = \sin 2\lambda' . \sin^2\tfrac{1}{2}C . \sec C$$

Here $\alpha = 1$, $\beta = \tan\lambda'$

$$\gamma = \sin 2\lambda' . \sin^2\tfrac{1}{2}C . \sec C, \quad \delta = 0$$

Hence $A = \frac{\beta}{\alpha} = \frac{1}{\sin 2\lambda' . \sin^2\frac{1}{2}C . \sec C}$

$$B = \frac{\gamma}{\alpha} - \frac{\beta . \delta^2}{\alpha^3} = \tan\lambda' . \sin^2 2\lambda' . \sin^4\tfrac{1}{2}C . \sec^2 C$$

wherefore,

$$d\lambda = \sin 2\lambda' . \sin^2\tfrac{1}{2}C . \sec C - \tan\lambda' . \sin^2 2\lambda' . \sin^4\tfrac{1}{2}C . \sec^2 C$$

which is the value of $d\lambda$ sufficiently correct for all practical purposes.-

The formula for $d\lambda$ & dC being obtained the linear values of the meridional and perpendecular arcs may be expressed thus.-

Linear value of the meridional arc Ab.-

$$r . \sin 1'' . (\lambda' \sim \lambda) \pm = r . \sin 2\lambda' . \sin^2\tfrac{1}{2}C . \sec C +$$

$$\gamma . \tan\lambda' . \sin^2 2\lambda' . \sin^4\tfrac{1}{2}C . \sec^2 C$$

in which the upper & lower = signs are to be used - according as B is to the north or South of A.- Also the term r is to be taken for the mean latitude between A & B

Linear value of the perpendicular arc Bb.-

$$\gamma . \sin 1'' . (I' \sim I) = 2\gamma . \sin^2\tfrac{1}{2}\lambda' . \tan C$$

$$+ \quad 2\gamma . \sin^4\tfrac{1}{2}\lambda' . \tan^3 C$$

Discrepancies between Vlacque's and Vega's Tables

ii. ó. ô	Sin.	9·2805988450	Vega
	"	9·2805988448	Vlacque
	tan.	9·2886522686	Vega
	"	9·2886522684	Vlacque
	Cot.	10·7113477314	Vega
	"	10·7113477315	Vlacque

Vega's values are used, because they are consistent amongst themselves.

Compared by
Sreenath Sirdar
& Shib narain Sutt

Hurilaong Meridional Series

Rules for a cursory examination of the Computation of a set of Circumpolar Star Observations

Compute the first registered N.P.D. and R of the circumpolar Star, and then seeing whether these elements are increasing or diminishing in the Nautical Almanac, examine whether the N.P.D^{s} and R^{s} as given in the Computation Sheets observe a similar law; and also examine that the change which has taken in either element is about the quantity from one consecutive elongation to another.-

The rule with reference to the examination of the elongations is this:- If the N.P.D. is increasing the elongation must increase.- Also if the N.P.D. is diminishing, the elongation must diminish.- Also the change in the elongation between two consecutive sets of observations is about the same quantity daily.- See that these circumstances obtain with reference to the elongations.-

The rule with reference to the examination of the Horary Angle is this:- When the N.P.D. is increasing, the horary angle must diminish.- Also when the N.P.D. is diminishing the horary angle must increase.- Also the change in the horary Angle from one consecutive elongation to another is about the same quantity.-

Compute the sidereal time of maximum Elongation in each case.-

Convert the sidereal time of each Max: elongation to chronometer time and with the aid of the observed chronometer time for each observation compute Δz by the Approximate formula.-

Also see whether the Chronometer is adjusted to Mean Solar Time. If it is then the Chronometer measure of the interval between two consecutive observations must be less than the sidereal measure of the same interval which circumstances examine if it has taken place